[CWMO 2011 question 4.] In a circle Γ_1 centred at O, AB and CD are two unequal chords intersecting at E inside Γ_1. A circle Γ_2 centred at I is tangent to Γ_1 internally at F, and also tangent to AB at G and CD at H. A line ℓ through O intersecting AB and CD at P and Q respectively such that $EP = EQ$. The line EF intersects ℓ at M. Prove that the line through M parallel to AB is tangent to Γ_1.

Solution. Let the extensions of FG and FH meet the circle Γ_1 at G' and H' respectively. Let h be the homothety cnetred at F mapping Γ_2 to Γ_1. Then $h(I) = O$, $h(G) = G'$ and $h(H) = H'$. As the tangent α to Γ_1 at G' is parallel to AB. it suffices to show α, ℓ and EF are concurrent at M.
Note that $\angle EPQ = \angle PQE = \angle GEI = \angle HEI$ so that IE is parallel OM. Since $h(I) = O$, we have $h(E) = M$. Then under the homothety h, the line AB goes to α, the line EF goes back to EF and the line IE goes to ℓ. Since lines AB, EF and IE concur at E, the lines α, EF and ℓ concur at M.