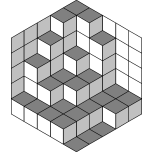




IMO 2007
Ha Noi, Vietnam



Day 1 - 25 July 2007

- 1 Real numbers a_1, a_2, \dots, a_n are given. For each i , ($1 \leq i \leq n$), define

$$d_i = \max\{a_j \mid 1 \leq j \leq i\} - \min\{a_j \mid i \leq j \leq n\}$$

and let $d = \max\{d_i \mid 1 \leq i \leq n\}$.

- (a) Prove that, for any real numbers $x_1 \leq x_2 \leq \dots \leq x_n$,

$$\max\{|x_i - a_i| \mid 1 \leq i \leq n\} \geq \frac{d}{2}. \quad (*)$$

- (b) Show that there are real numbers $x_1 \leq x_2 \leq \dots \leq x_n$ such that the equality holds in (*).

- 2 Consider five points A, B, C, D and E such that $ABCD$ is a parallelogram and $BCED$ is a cyclic quadrilateral. Let ℓ be a line passing through A . Suppose that ℓ intersects the interior of the segment DC at F and intersects line BC at G . Suppose also that $EF = EG = EC$. Prove that ℓ is the bisector of angle DAB .

- 3 In a mathematical competition some competitors are friends. Friendship is always mutual. Call a group of competitors a *clique* if each two of them are friends. (In particular, any group of fewer than two competitors is a clique.) The number of members of a clique is called its *size*.

Given that, in this competition, the largest size of a clique is even, prove that the competitors can be arranged into two rooms such that the largest size of a clique contained in one room is the same as the largest size of a clique contained in the other room.