In triangle $ABC$ the bisector of angle $BCA$ intersects the circumcircle again at $R$, the perpendicular bisector of $BC$ at $P$, and the perpendicular bisector of $AC$ at $Q$. The midpoint of $BC$ is $K$ and the midpoint of $AC$ is $L$. Prove that the triangles $RPK$ and $RQL$ have the same area.

Let $a$ and $b$ be positive integers. Show that if $4ab - 1$ divides $(4a^2 - 1)^2$, then $a = b$.

Let $n$ be a positive integer. Consider

$$S = \{(x,y,z) \mid x,y,z \in \{0,1,\ldots,n\}, x+y+z > 0\}$$

as a set of $(n+1)^3 - 1$ points in the three-dimensional space. Determine the smallest possible number of planes, the union of which contains $S$ but does not include $(0,0,0)$. 