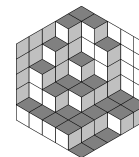




IMO 2007  
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Day 2 - 26 July 2007

- 4 In triangle  $ABC$  the bisector of angle  $BCA$  intersects the circumcircle again at  $R$ , the perpendicular bisector of  $BC$  at  $P$ , and the perpendicular bisector of  $AC$  at  $Q$ . The midpoint of  $BC$  is  $K$  and the midpoint of  $AC$  is  $L$ . Prove that the triangles  $RPK$  and  $RQL$  have the same area.
- 5 Let  $a$  and  $b$  be positive integers. Show that if  $4ab - 1$  divides  $(4a^2 - 1)^2$ , then  $a = b$ .
- 6 Let  $n$  be a positive integer. Consider

$$S = \{(x, y, z) \mid x, y, z \in \{0, 1, \dots, n\}, x + y + z > 0\}$$

as a set of  $(n + 1)^3 - 1$  points in the three-dimensional space. Determine the smallest possible number of planes, the union of which contains  $S$  but does not include  $(0, 0, 0)$ .