Day 2 - *26 July 2007*

$\boxed{4}$ In triangle ABC the bisector of angle BCA intersects the circumcircle again at R, the perpendicular bisector of BC at P, and the perpendicular bisector of AC at Q. The midpoint of BC is K and the midpoint of AC is L. Prove that the triangles RPK and RQL have the same area.

$\boxed{5}$ Let a and b be positive integers. Show that if $4ab-1$ divides $(4a^2-1)^2$, then $a=b$.

$\boxed{6}$ Let n be a positive integer. Consider

$$S = \{(x,y,z) \mid x,y,z \in \{0,1,\ldots,n\}, x+y+z>0\}$$

as a set of $(n+1)^3-1$ points in the three-dimensional space. Determine the smallest possible number of planes, the union of which contains S but does not include $(0,0,0)$.