Day 1

1. Let $a_1, a_2, \ldots, a_{41} \in \mathbb{R}$, such that $a_{41} = a_1, \sum_{i=1}^{40} a_i = 0$, and for any $i = 1, 2, \ldots, 40$, $|a_i - a_{i+1}| \leq 1$. Determine the greatest possible value of

(a) $a_{10} + a_{20} + a_{30} + a_{40}$;
(b) $a_{10} \cdot a_{20} + a_{30} \cdot a_{40}$.

2. In triangle $ABC$, $AB > AC$. The bisector of $\angle BAC$ meets $BC$ at $D$. $P$ is on line $DA$, such that $A$ lies between $P$ and $D$. $PQ$ is tangent to $\odot(ABD)$ at $Q$. $PR$ is tangent to $\odot(ACD)$ at $R$. $CQ$ meets $BR$ at $K$. The line parallel to $BC$ and passing through $K$ meets $QD, AD, RD$ at $E, L, F$, respectively. Prove that $EL = KF$.

3. Let $S$ be a set, $|S| = 35$, $F = \{ f : S \to S \}$, called $F$ satisfying $P(k)$, if for any $x, y \in S$, there exist $f_1, \ldots, f_k$ (can be equiv), such that $f_k(f_{k-1}(\cdots(f_1(x)))) = f_k(f_{k-1}(\cdots(f_1(y))))$. Find the minimum value of $m$, if $F$ satisfy $P(2019)$, then it satisfy $P(m)$.

Day 2

1. Find the largest positive constant $C$ such that the following is satisfied: Given $n$ arcs (containing their endpoints) $A_1, A_2, \ldots, A_n$ on the circumference of a circle, where among all sets of three arcs $(A_i, A_j, A_k)$ $(1 \leq i < j < k \leq n)$, at least half of them has $A_i \cap A_j \cap A_k$ nonempty, then there exists $l > Cn$, such that we can choose $l$ arcs among $A_1, A_2, \ldots, A_n$, whose intersection is nonempty.

2. Given any positive integer $c$, denote $p(c)$ as the largest prime factor of $c$. A sequence $\{a_n\}$ of positive integers satisfies $a_1 > 1$ and $a_{n+1} = a_n + p(a_n)$ for all $n \geq 2$. Prove that there must exist at least one perfect square in sequence $\{a_n\}$.

3. Does there exist positive reals $a_0, a_1, \ldots, a_{19}$, such that the polynomial $P(x) = x^{20} + a_{19}x^{19} + \ldots + a_1x + a_0$ does not have any real roots, yet all polynomials formed from swapping any two coefficients $a_i, a_j$ has at least one real root?