1. Let \( n \) be a given positive integer, and \( x_1, x_2, \ldots, x_n \) are real numbers such that the sum \( \sum_{k=1}^{n} x_k \) is an integer. Let \( d_k = \min_{m \in \mathbb{Z}} |x_k - m|, 1 \leq k \leq n \). Determine, with proof, the maximum value of the sum \( \sum_{k=1}^{n} d_k \).

2. As shown in the figure, circles \( \omega_1 \) and \( \omega_2 \) are tangent to each other at the point \( T \). \( M, N \) are two distinct points on \( \omega_1 \) and different from \( T \). \( AB \) and \( CD \) are two chords of \( \omega_2 \) passing \( M, N \) respectively. Prove that if the segments \( AC, BD, MN \) meet at the same point \( K \), then the line \( TK \) bisects \( \angle MTN \).

3. Let \( n \geq 2 \), be an integer, and \( x_1, x_2, \ldots, x_n \) are positive real numbers such that \( \sum_{i=1}^{n} x_i = 1 \). Prove that

\[
\left( \sum_{i=1}^{n} \frac{1}{1-x_i} \right) \left( \sum_{1 \leq i < j \leq n} x_i x_j \right) \leq \frac{n}{2}.
\]

4. For 100 straight lines on a plane, let \( T \) be the set of all right-angled triangles bounded by some 3 lines. Determine, with proof, the maximum value of \( |T| \).