Problem 4. Consider the sequence \( a_1, a_2, \ldots \) defined by

\[
a_n = 2^n + 3^n + 6^n - 1 \quad (n = 1, 2, \ldots).
\]

Determine all positive integers that are relatively prime to every term of the sequence.

Problem 5. Let \( ABCD \) be a given convex quadrilateral with sides \( BC \) and \( AD \) equal in length and not parallel. Let \( E \) and \( F \) be interior points of the sides \( BC \) and \( AD \) respectively such that \( BE = DF \). The lines \( AC \) and \( BD \) meet at \( P \), the lines \( BD \) and \( EF \) meet at \( Q \), the lines \( EF \) and \( AC \) meet at \( R \). Consider all the triangles \( PQR \) as \( E \) and \( F \) vary. Show that the circumcircles of these triangles have a common point other than \( P \).

Problem 6. In a mathematical competition 6 problems were posed to the contestants. Each pair of problems was solved by more than \( \frac{2}{3} \) of the contestants. Nobody solved all 6 problems. Show that there were at least 2 contestants who each solved exactly 5 problems.