Thursday, July 8, 2010

**Problem 4.** Let $P$ be a point inside the triangle $ABC$. The lines $AP$, $BP$ and $CP$ intersect the circumcircle $\Gamma$ of triangle $ABC$ again at the points $K$, $L$ and $M$ respectively. The tangent to $\Gamma$ at $C$ intersects the line $AB$ at $S$. Suppose that $SC = SP$. Prove that $MK = ML$.

**Problem 5.** In each of six boxes $B_1, B_2, B_3, B_4, B_5, B_6$ there is initially one coin. There are two types of operation allowed:

- *Type 1:* Choose a nonempty box $B_j$ with $1 \leq j \leq 5$. Remove one coin from $B_j$ and add two coins to $B_{j+1}$.
- *Type 2:* Choose a nonempty box $B_k$ with $1 \leq k \leq 4$. Remove one coin from $B_k$ and exchange the contents of (possibly empty) boxes $B_{k+1}$ and $B_{k+2}$.

Determine whether there is a finite sequence of such operations that results in boxes $B_1, B_2, B_3, B_4, B_5$ being empty and box $B_6$ containing exactly $2010^{2010}$ coins. (Note that $a^{bc} = a^{(bc)}$.)

**Problem 6.** Let $a_1, a_2, a_3, \ldots$ be a sequence of positive real numbers. Suppose that for some positive integer $s$, we have

$$a_n = \max\{a_k + a_{n-k} \mid 1 \leq k \leq n - 1\}$$

for all $n > s$. Prove that there exist positive integers $\ell$ and $N$, with $\ell \leq s$ and such that $a_n = a_\ell + a_{n-\ell}$ for all $n \geq N$. 

*Language: English*  
*Time: 4 hours 30 minutes.*  
*Each problem is worth 7 marks.*