<u>9th Singapore Mathematics Symposium</u>

Date: 28 September, 2018 (Friday)

Venue: National University of Singapore, LT34 (in Block S17, Mathematics Department building)

Time: 1pm – 5:45pm

Schedule:

1:00 – 1:05: Welcome by Victor Tan, SMS president

1:05 – 1:55: Tan Ser Peow (NUS Math): Hyperbolic jigsaws and families of pseudo-modular surfaces.

2:05 – 2:55: Kartik Natarajan (SUTD Engineering Systems and Design): On The Interplay of Optimization and Probability in Decision Making

2:55 – 3:25: Tea break

3:25 – 4:15: Bernhard Schmidt (NTU SPMS):

4:25 – 5:15: Zhang De Qi (NUS Math): Geometric structures of algebraic manifolds – **MMP**, **Abundance**, **BAB** conjectures

5:25 – 5:45: Poster Prize Presentation and Closing Remarks

Organizing Committee: Gan Wee Teck (NUS Math), Chan Song Heng (NTU SPMS) and Adrian Roellin (NUS DSAP)

<u>Titles and Abstracts:</u>

(1) Speaker: Prof. Tan Ser Peow (NUS Math)

Title: Hyperbolic jigsaws and families of pseudo-modular surfaces.

Abstract: It is well known that the set of cusps of the modular group PSL(2,Z) is the set of rationals including infinity, this follows from the well-known Euclidean algorithm. In general, determining the set of cusps of a given Fuchsian group (discrete subgroup of PSL(2,R)) is a difficult question and not many families of examples are known where the set of cusps is completely determined. Commensurable groups have the same cusp set, and the natural question is whether the converse holds: does the cusp set determine the commensurability class. Long and Reid defined a pseudo-modular group to be a Fuchsian group which is NOT commensurable with the modular group, but which has cusp set all of the rationals including infinity. The corresponding surface is called a pseudo-modular surface. Long and Reid showed the rather surprising result that such surfaces existed by constructing a small number of examples of pseudo-modular groups which belong to different commensurability classes (so that the cusp set does not determine the commensurability class) and asked (the natural question) if there are infinitely many commensurability classes of pseudo-modular surfaces. In this talk, we will show how to construct such infinite families by introducing a general construction of surfaces whose fundamental domains are obtained by gluing together marked ideal triangular tiles, which we call hyperbolic jigsaw surfaces. In the case of jigsaw surfaces made up of the two simplest tiles, we show that there is a pseudo-Euclidean algorithm associated to the groups which brings every rational to infinity, so that all such surfaces are indeed pseudo-modular. This is joint work with Beicheng Lou and Anh Duc Vo.

(2) Speaker: Prof. Kartik Natarajan (SUTD Engineering Systems and Design)

Title: On The Interplay of Optimization and Probability in Decision Making

Abstract: Decision making under uncertainty is an important problem that shows up in many practical applications. There are some fundamental challenges in efficiently solving these problems, primarily due to the interplay of optimization and probability. The past decade has seen significant interest in “distributionally robust optimization” where optimal decisions are prescribed for the worst-case distribution in an appropriately defined ambiguity set. In this talk, I will review some of the key ideas in this approach driven by new applications and developments and highlight where I think research opportunities lie.

(3) Speaker: Prof. Bernhard Schmidt (NTU, School of Physical and Mathematical Sciences)

Title: Bilinear forms on finite abelian groups and Butson matrices

Abstract: Bilinear forms over finite fields are well understood and have been used for the construction of numerous combinatorial objects such as combinatorial designs and substrutures of finite geometries. There exists a theory of bilinear forms on finite abelian groups, too, but their applications to combinatorics, except for the special case of forms on additive groups of finite fields, are rare. We will show how that any symmetric and nondegenerate bilinear form on a finite abelian group can be used to construct Butson matrices. Here, by a Butson matrix, we mean a square matrix whose entries are complex roots of unity and whose rows are pairwise orthogonal with respect to the standard Hermitian inner product. This is joint work with Tai Do Duc.

(4) Speaker: Prof. Zhang De Qi (NUS Department of Mathematics)

Title: Geometric structures of algebraic manifolds – **MMP**, **Abundance**, **BAB** conjectures

Abstract: This talk reports the exciting new developments in birational geometry: the **BAB** conjecture solved by Birkar and the **Iitaka conjecture** solved by Birkar and myself.

An algebraic manifold **X** is the common solution set of several polynomial equations in variables **x_i**'s. It has a natural geometric structure as a submanifold of the projective space **P^n** in coordinates **x_i**'s. The minimal model program (**MMP**) aims to find a better model **X'** birational (i.e., generically isomorphic) to **X** with a better structure.

Minimal model conjecture = **MMC** (Existence and Abundance).

(1) Either the cotangent line bundle **$K_{X'}$** is positive; or

(2) There is a **Fano** or **Iitaka** fibration

g: X' ——> Y

to a lower dimensional manifold **Y**, such that the general fibre **F = X_y** lying over a general point **y** in **Y**, has its cotangent line bundle

(2a) **K_F** being negative (such **F** is called a **Fano** variety); or

(2b) **K_F** being trivial (such **F** is called a **Calabi - Yau** variety).

The **MMC** is known in dimension at most three, due to **Mori** et al, [Mori88].

The **Iitaka conjecture** asserts that the Iitaka fibration **g: X' ——> Y** above is given by the sections of pluri-cotangent line bundle **mK_F** for some bounded m. This is solved in our joint work [BZ14], by developing the generalized MMP (**GMMP**).

The **Borisov - Alexseev - Borisov** (**BAB**) conjecture asserts that Fano varieties with mild singularities form a bounded family. This has been spectacularly proved by Caucher Birkar [B16a, b] by making use of **GMMP**, Shokurov's complements, …,thus earning him the Fields medal in this August 2018.

The **boundedness conjecture** of Calabi-Yau varieties (= Miles Reid's fantasy) is still open.

The **abundance conjecture** asserting that the meromorphic map **g: X' ——> Y** above is everywhere well-defined, holomorphic, is still open in dimension four or above.

Main references:

Mori88, Mori, JAMS (1988)
BCHM2010, Birkar-Cascini-Hacon-McKernan, JAMS (2010)
BZ14, Birkar-Zhang, arXiv:1410.0938 = IHES (2016)
B16a, Birkar, arXiv:1603.05765
B16b, Birkar, arXiv:1609.05543