9th Singapore Mathematics Symposium

Date: 28 September, 2018 (Friday)

Venue: National University of Singapore, LT34 (in Block S17, Mathematics Department building)

Time: 1pm – 5:45pm

Schedule:

1:00 – 1:05: Welcome by Victor Tan, SMS president


2:55 – 3:25: Tea break

3:25 – 4:15: Bernhard Schmidt (NTU SPMS):


5:25 – 5:45: Poster Prize Presentation and Closing Remarks

Organizing Committee: Gan Wee Teck (NUS Math), Chan Song Heng (NTU SPMS) and Adrian Roellin (NUS DSAP)

Titles and Abstracts:

(1) Speaker: Prof. Tan Ser Peow (NUS Math)

Title: Hyperbolic jigsaws and families of pseudo-modular surfaces.

Abstract: It is well known that the set of cusps of the modular group PSL(2,Z) is the set of rationals including infinity, this follows from the well-known Euclidean algorithm. In general, determining the set of cusps of a given Fuchsian group (discrete subgroup of PSL(2,R)) is a difficult question and not many families of examples are known where the set of cusps is completely determined. Commensurable groups have the same cusp set, and the natural question is whether the converse holds: does the cusp set determine the commensurability class. Long and Reid defined a pseudo-modular group to be a Fuchsian group which is NOT commensurable with the modular group, but which has cusp set all of
the rationals including infinity. The corresponding surface is called a pseudo-modular
surface. Long and Reid showed the rather surprising result that such surfaces existed by
constructing a small number of examples of pseudo-modular groups which belong to
different commensurability classes (so that the cusp set does not determine the
commensurability class) and asked (the natural question) if there are infinitely many
commensurability classes of pseudo-modular surfaces. In this talk, we will show how to
construct such infinite families by introducing a general construction of surfaces whose
fundamental domains are obtained by gluing together marked ideal triangular tiles, which
we call hyperbolic jigsaw surfaces. In the case of jigsaw surfaces made up of the two
simplest tiles, we show that there is a pseudo-Euclidean algorithm associated to the
groups which brings every rational to infinity, so that all such surfaces are indeed pseudo-
modular. This is joint work with Beicheng Lou and Anh Duc Vo.

(2) Speaker: Prof. Kartik Natarajan (SUTD Engineering Systems and Design)

Title: On The Interplay of Optimization and Probability in Decision Making

Abstract: Decision making under uncertainty is an important problem that shows up in
many practical applications. There are some fundamental challenges in efficiently solving
these problems, primarily due to the interplay of optimization and probability. The past
decade has seen significant interest in “distributionally robust optimization” where
optimal decisions are prescribed for the worst-case distribution in an appropriately
defined ambiguity set. In this talk, I will review some of the key ideas in this
approach driven by new applications and developments and highlight where I think
research opportunities lie.

(3) Speaker: Prof. Bernhard Schmidt (NTU, School of Physical and Mathematical Sciences)

Title: Bilinear forms on finite abelian groups and Butson matrices

Abstract: Bilinear forms over finite fields are well understood and have been used for the
construction of numerous combinatorial objects such as combinatorial designs and
substrutures of finite geometries. There exists a theory of bilinear forms on finite abelian
groups, too, but their applications to combinatorics, except for the special case of forms
on additive groups of finite fields, are rare. We will show how that any symmetric and
nondegenerate bilinear form on a finite abelian group can be used to construct Butson
matrices. Here, by a Butson matrix, we mean a square matrix whose entries are complex
roots of unity and whose rows are pairwise orthogonal with respect to the standard
Hermitian inner product. This is joint work with Tai Do Duc.

(4) Speaker: Prof. Zhang De Qi (NUS Department of Mathematics)

Title: Geometric structures of algebraic manifolds – MMP, Abundance, BAB conjectures

Abstract: This talk reports the exciting new developments in birational geometry: the
BAB conjecture solved by Birkar and the Iitaka conjecture solved by Birkar and myself.
An algebraic manifold $X$ is the common solution set of several polynomial equations in variables $x_i$'s. It has a natural geometric structure as a submanifold of the projective space $\mathbb{P}^n$ in coordinates $x_i$'s. The minimal model program (MMP) aims to find a better model $X'$ birational (i.e., generically isomorphic) to $X$ with a better structure.

**Minimal model conjecture** = MMC (Existence and Abundance).

1. Either the cotangent line bundle $K_X$ is positive; or
2. There is a Fano or Iitaka fibration
   \[ g: X' \longrightarrow Y \]
   to a lower dimensional manifold $Y$, such that the general fibre $F = X_y$ lying over a general point $y$ in $Y$, has its cotangent line bundle
   \[(2a)\ K_F\ being\ negative\ (such\ F\ is\ called\ a\ Fano\ variety);\ or\]
   \[(2b)\ K_F\ being\ trivial\ (such\ F\ is\ called\ a\ Calabi - Yau\ variety).\]

The MMC is known in dimension at most three, due to Mori et al, [Mori88].

The **Iitaka conjecture** asserts that the Iitaka fibration $g: X' \longrightarrow Y$ above is given by the sections of pluri-cotangent line bundle $mK_F$ for some bounded $m$. This is solved in our joint work [BZ14], by developing the generalized MMP (GMMP).

The **Borisov - Alexseev - Borisov (BAB)** conjecture asserts that Fano varieties with mild singularities form a bounded family. This has been spectacularly proved by Caucher Birkar [B16a, b] by making use of GMMP, Shokurov's complements, ..., thus earning him the Fields medal in this August 2018.

The **boundedness conjecture** of Calabi-Yau varieties (= Miles Reid’s fantasy) is still open.

The **abundance conjecture** asserting that the meromorphic map $g: X' \longrightarrow Y$ above is everywhere well-defined, holomorphic, is still open in dimension four or above.

**Main references**:

- Mori88, Mori, JAMS (1988)
- B16a, Birkar, arXiv:1603.05765
- B16b, Birkar, arXiv:1609.05543