

## The Tenth Singapore Mathematics Symposium

**Date:** 27 September, 2019 (Friday)

**Venue:** NTU School of Physical and Mathematical Sciences, LT4

**Time:** 2pm – 5:45pm

### Schedule:

2:00 – 2:05: Welcome by Victor Tan, SMS president

2:05 – 2:55: Vincent Y.F. Tan (NUS ECE & NUS Math): Analysis of Optimization Algorithms via Sum-of-Squares

3:05 – 3:55: Lim Nengli (SUTD ESD): Rough paths theory and Ito-Stratonovich formulas for general Gaussian processes

3:55 - 4:25: Tea break

4:25 – 5:15: Wu Guohua (NTU SPMS): Doing algebra in a reverse direction

5:15 – 5:45: Poster Prize Presentation and Closing Remarks

**Organizing Committee:** Nicolas Privault (Chair) (NTU SPMS), Gan Wee Teck (NUS Math), and Ng Keng Meng (NTU SPMS)

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### Titles and Abstracts

#### **Analysis of Optimization Algorithms via Sum-of-Squares**

By Vincent Y.F. Tan (NUS ECE & NUS Math)

**Abstract:** In this work, we introduce a new framework for unifying and systematizing the performance analysis of first-order black-box optimization algorithms for unconstrained convex minimization over finite-dimensional Euclidean spaces. The low-cost iteration complexity enjoyed by this class of algorithms renders them particularly relevant for applications in machine learning and large-scale data analysis. However, existing proofs of convergence of such optimization algorithms consist mostly of ad-hoc arguments and case-by-case analyses. On the other hand, our approach is based on sum-of-squares optimization and puts forward a promising framework for unifying the convergence analyses of optimization algorithms. Illustrating the usefulness of our approach, we recover several known convergence bounds for four widely-used first order algorithms in a unified manner, and also derive one new convergence result for gradient descent with Armijo-terminated line search.

This is joint work with Sandra S. Y. Tan and Antonios Varvitsiotis and the preprint can be found here <https://arxiv.org/abs/1906.04648>.

## **Rough paths theory and Ito-Stratonovich formulas for general Gaussian processes**

By Lim Nengli (SUTD ESD)

**Abstract:** Using martingale convergence, the classical theory of stochastic integration bypasses the issues of integrating against signals with unbounded variation. We use this as a starting point to motivate the discussion of rough paths theory, which extends the integration framework beyond martingales by augmenting such rough signals with their higher-order iterated integrals. In particular, we show that one can extend the classical Ito-Stratonovich formulas to general Gaussian processes such as fractional Brownian motion.

## **Doing algebra in a reverse direction**

By Wu Guohua (NTU SPMS)

**Abstract:** In 1931, Kurt Gödel published his incompleteness theorems of mathematical logic, revealing the inherent limitations of formal axiomatic systems which are capable of modelling basic arithmetic. In some sense, these incompleteness theorems can be interpreted as showing that Hilbert's program cannot be realized.

In this talk, we first describe basic ideas of working with axiomatic systems, Hilbert's program, and Gödel's incompleteness theorems. We will then talk about a program called reverse mathematics, a continuation of Hilbert's program, which has been initiated and developed by Harvey Friedman and Steven Simpson, and use examples from algebra to demonstrate the idea of doing research in reverse algebra.