## Problem 2 (Heuristics)

**Find all two-digit number *N* such that *N* + 2 is also a two-digit number and the digit sum of *N* + 2 is less than the digit sum of *N*.**

**Understanding the problem**

**Think of say the number *N* = 12. Digit sum = 1 + 2 = 3**

**Then *N* + 2 = 14. Digit sum = 1 + 4 = 5. (Not good)**

**What if *N* = 24? Digit sum = 6, *N* + 2 = 26 & digit sum = 8**

**Conclusion: Tens digit cannot be smaller than unit digit?**

**Not true! For example, *N* = 21.**

## Problem 2 (Heuristics)

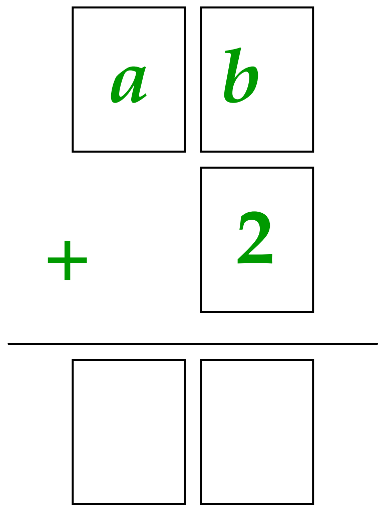
**Strategy 1: Run through all the 2-digit numbers From 10 to 99.** ***Easy but tedious.***

# Problem 2 (Heuristics)

**Strategy 2:**

**Digit sum of $N$ is $a + b$**

**For digit sum of $N + 2$ to be less $a + b$, the unit digit of $N + 2$ must be such that $b + 2$ Must be greater than or equal to 10. Therefore $b = 8$ or 9**




**When $b = 8$, $a$ can be any digits from 1 to 9.**
**When $b = 9$, $a$ can be any digits from 1 to 9 too.**

**$\therefore N = 18, 28, 38, 48, \ldots, 98, 19, 29, 39, \ldots, 99$**

## Background for Problem 3

$2 = 1 \times 2$ factors are 1, 2

$4 = 1 \times 4 = 2 \times 2$ factors are 1, 2, 4 $\quad 4 = 2^2$

$6 = 1 \times 6 = 2 \times 3$ factors are 1, 2, 3, 6 $\quad 6 = 2 \times 3$

$9 = 1 \times 9 = 3 \times 3$ factors are 1, 3, 9 $\quad 9 = 3^2$

$10 = 1 \times 10 = 2 \times 5$ factors are 1, 2, 5, 10 $\quad 10 = 2 \times 5$

$11 = 1 \times 11$ factors are 1, 11

$12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ factors are 1, 2, 3, 4, 12

$12 = 2^2 \times 3$

# Background for Problem 3

$24 = 2^3 \times 3^1$

Factors are 1, 2, 3, 4, 6, 8, 12, 24

| | |
|---|---|
| $1 = 2^0 \times 3^0$ | $6 = 2^1 \times 3^1$ |
| $2 = 2^1 \times 3^0$ | $8 = 2^3 \times 3^0$ |
| $3 = 2^0 \times 3^1$ | $12 = 2^2 \times 3^0$ |
| $4 = 2^2 \times 3^0$ | $24 = 2^3 \times 3^1$ |

Do you observe any pattern?

The number of factors is given by $(3+1)(1+1) = 8$

## Problem 3

**Find the largest four-digit number having exactly three factors, including 1 and itself**

**Hence, numbers with exactly three factors must be from $p^2$.**

**The largest prime number $p$ such that $p^2 \leq 9999$ is 97.**

**$\therefore$ This largest 4-digit number is $97^2 = 9409$**

# Problem 4 (Heuristics)

**Find the largest three-digit number which has exactly ten factors, including 1 and itself**

**Hint: $10 = 2 \times 5$**

**The number must be of the form $p \times q^4$, $p$ and $q$ are prime numbers.**

**Check systematically to arrive at the answer.**

## Problem 4 (Heuristics)

| $p$ | $q$ | $q^4$ | $p \times q^4$ |
|---|---|---|---|
| 2 | 5 | 625 | 1250 |
| 11 | 3 | 81 | 891 |
| 13 | 3 | 81 | 1053 |
| 61 | 2 | 16 | 976 |
| 67 | 2 | 16 | 1072 |

# Problem 5

**How many times must 2 dice be thrown to be sure that the same total occurs at least six times?**

**Hint:**

**The possible sum are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12**

**Answer: $5 \times 11 + 1 = 56$**

# Problem 5

**How many times must 2 dice be thrown to be sure that the same total occurs at least six times?**

**Hint:**

**The possible sum are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12**

**Answer: $5 \times 11 + 1 = 56$**

# Problem 6

**How many times must $n$ dice be thrown to be sure that the same total occurs at least $p$ times?**

**Answer: $(6n-n+1)(p-1)+1=(5n+1)(p-1)+1$**

# Problem 7

**How many people must there be, at least, in a room so that there will be at least 2 people with the same birthday?**

**Answer: 366+1 = 367**

# Problem 7

**How many people must there be, at least, in room so that there will be at least 2 people with the same birthday?**

**What would be the minimum number of people needed for at least 5 people with the same birthday?**

**Anwer: $366 \times 4 + 1$**

# Understanding the problem

**Prime numbers:**

**2, 3, 5, 7, 11, 13, 17, 19,…**

**Fractions of the form** ***(p - 1)/p*** **:**

**6/7, 10/11, 12/13, 16/17, 18/19,…**

**Units fractions:**

**1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9**

**Why** ***p*** **need to be more than 3?**

# Understanding the problem

**Possible sum of unit fractions:**

**1/2 + 1/3 = 5/6    1/2 + 1/4 = 3/4    1/3+1/4 = 7/12**

**Facts:**

**1/2 > 1/3 > 1/4 > 1/5 > 1/6 > 1/7 > 1/8 > 1/9 >…**

**Hence,**

**1/2 < 2/3 < 3/4 < 4/5 < 5/6 < 6/7 < 7/8 < 8/9 <…**

**Why?**