

SMS Project Workshop (September 9, 2006)

Supplementary Material
from Prof Koh Khee Meng's Talk

Some Relevant Math Magazines

1. Arithmetic Teacher
2. Teaching Children Mathematics
3. Mathematics Teaching
4. Teaching and Learning (NIE)
5. Focus on Learning Problems in Mathematics
6. Mathematics Teacher
7. Mathematics Teaching in the Middle School
8. Mathematics in School
9. The Mathematics Educator (NIE)

The website <http://www.nctm.org/> is also helpful.

PYTHAGORAS'

THEOREM REVISITED

Reproduced from Singapore Scientist, Vol 14 No. 4, 1989

This celebrated theorem is one of the most important in all of mathematics and is formally known as Proposition 47 in Euclid's Elements Book I. Although knowledge of its application was known long before his time, it was undoubtedly Pythagoras who generalised it from the Egyptian's empirical rule ($3^2 + 4^2 = 5^2$) and first demonstrated it about 540 B.C.

The Egyptians realised as early as 2000 B.C. that $3^2 + 4^2 = 5^2$ but there is no evidence that they knew or could prove the right-angle property of the figure involved. Even better were the Babylonians, who apparently knew the theorem a millennium before Pythagoras. A clay tablet Plimpton 322, dating from 1900 to 1600 B.C. contains at least 15 different sets of Pythagorean triples (whole numbers a, b, c which satisfy $a^2 + b^2 = c^2$).

The Chinese also had knowledge of the result as indicated in the Chou Pei Suan Ching, which probably dates before the Han Dynasty (202 B.C.) However, no actual proof of the theorem was demonstrated.

PYTHAGORAS'

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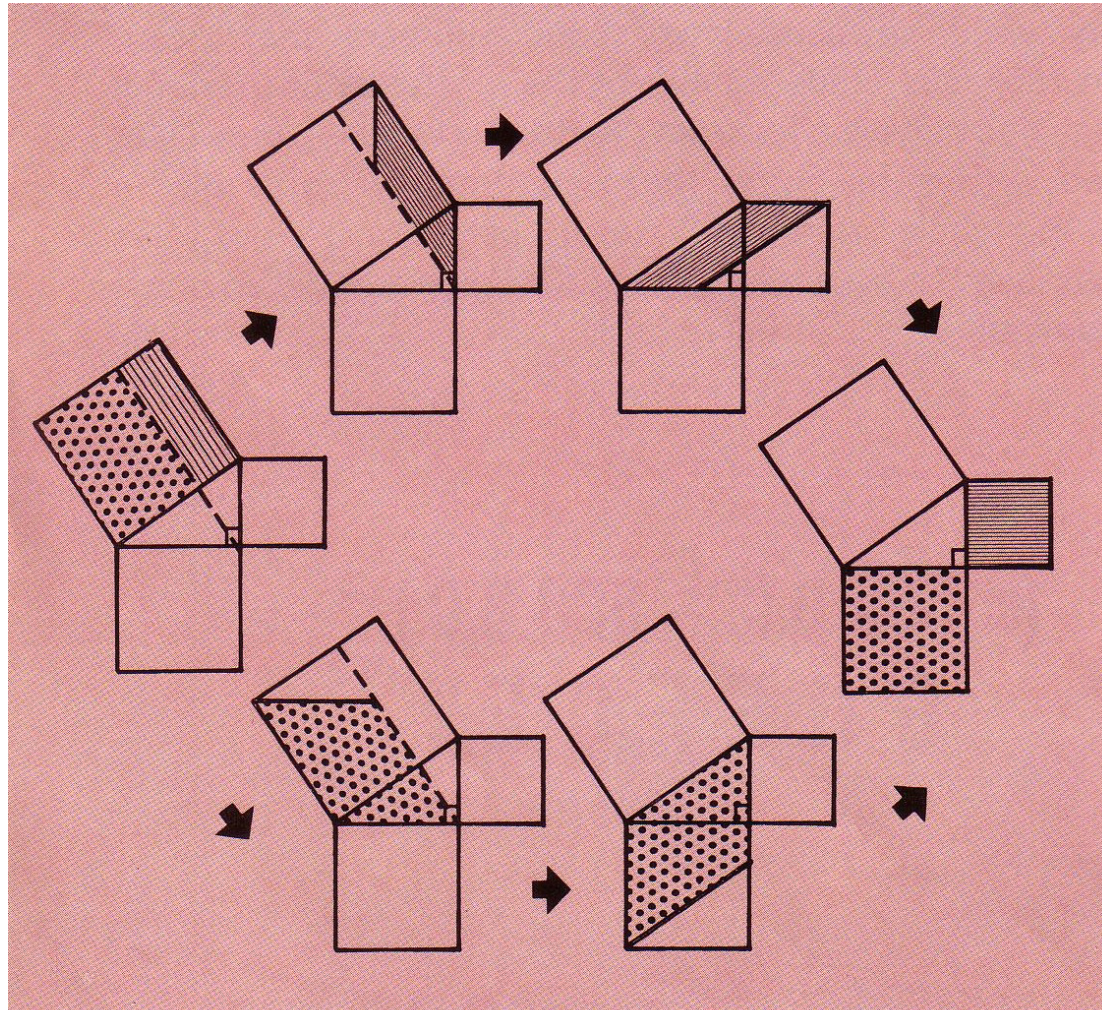
The Hindu Sulvasutras (dating back to 5 centuries before Christ) also carried an implication of the theorem, but again there is no evidence that they had any notion of a proof.

Pythagoras himself probably proved the theorem using the proportionality of similar figures. Later, with the realisation of the existence of irrational numbers and incommensurable lengths, his proof became somewhat invalid and a more adequate one was needed by the time of Euclid's Elements (c. 300 B.C.) Euclid himself was credited with the proof of his Proposition I, 47, which became the classical standard. Nevertheless, through the ages, literally hundreds of other proofs have emerged, making the Pythagorean Proposition perhaps the most widely proved theorem in mathematics.

Here are some of the proofs of Pythagoras' Theorem. How many do you know?

Proof of Pythagoras Theorem

Method 1

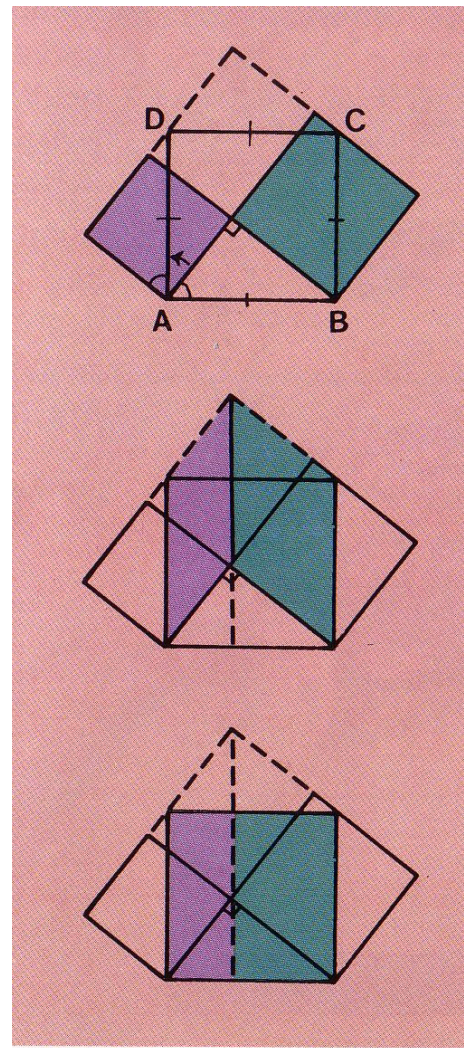


1. Modern version of the standard Euclidean proof using area-preserving transformations — shears and rotations.

Proof of Pythagoras Theorem Method 2

2. (Convince yourself first that $ABCD$ is a square.)

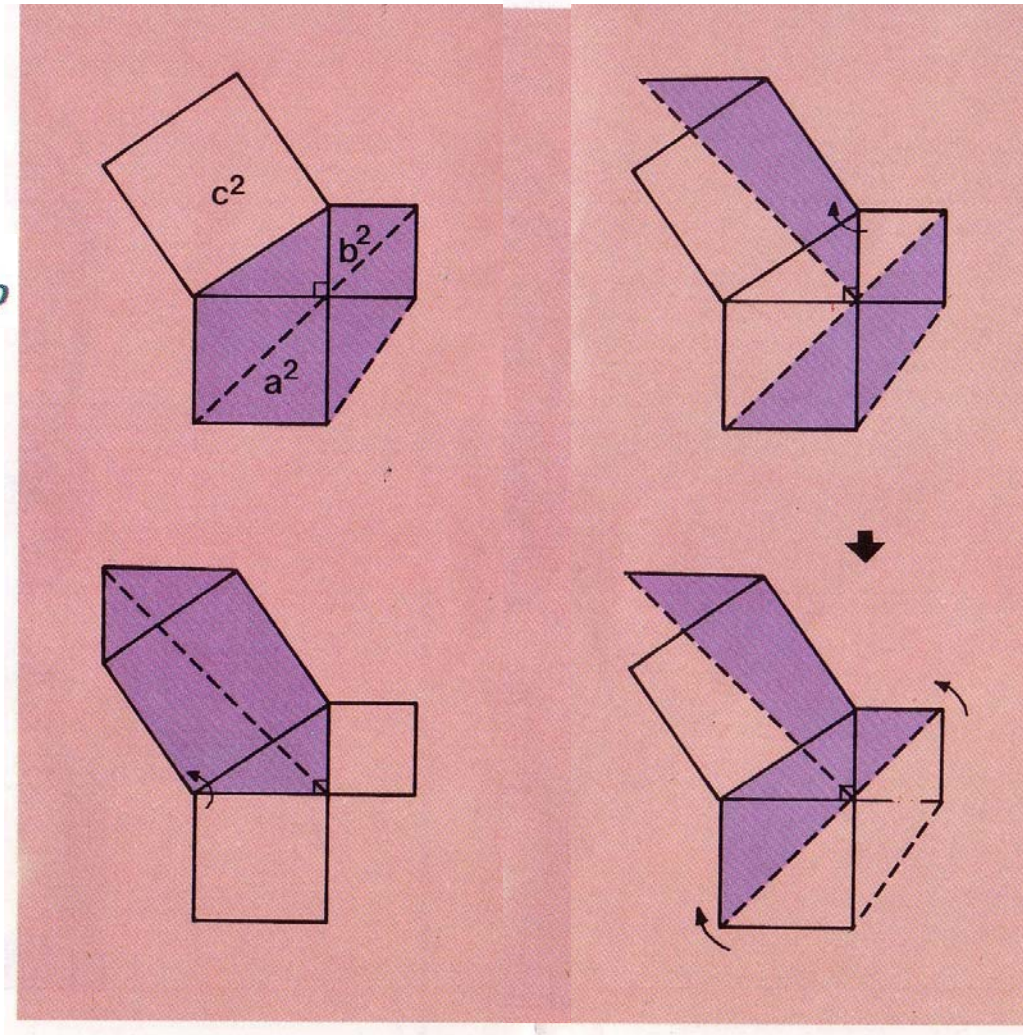
Another transformational proof using, very economically, only area-preserving shears.



Proof of Pythagoras Theorem

Method 3

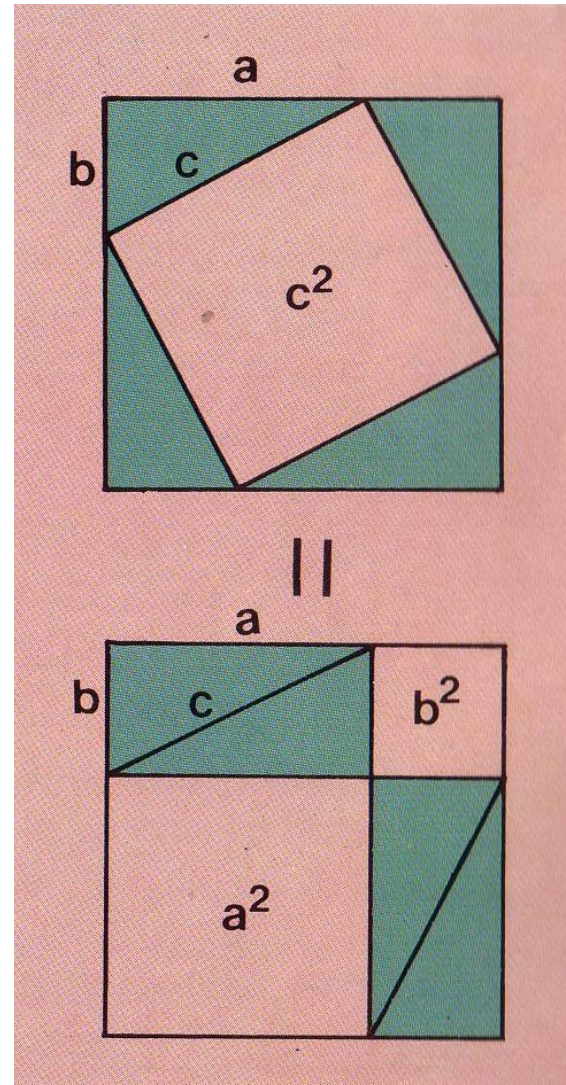
3. *Modern interpretation of a proof due to Leonardo da Vinci, 1452-1519.*



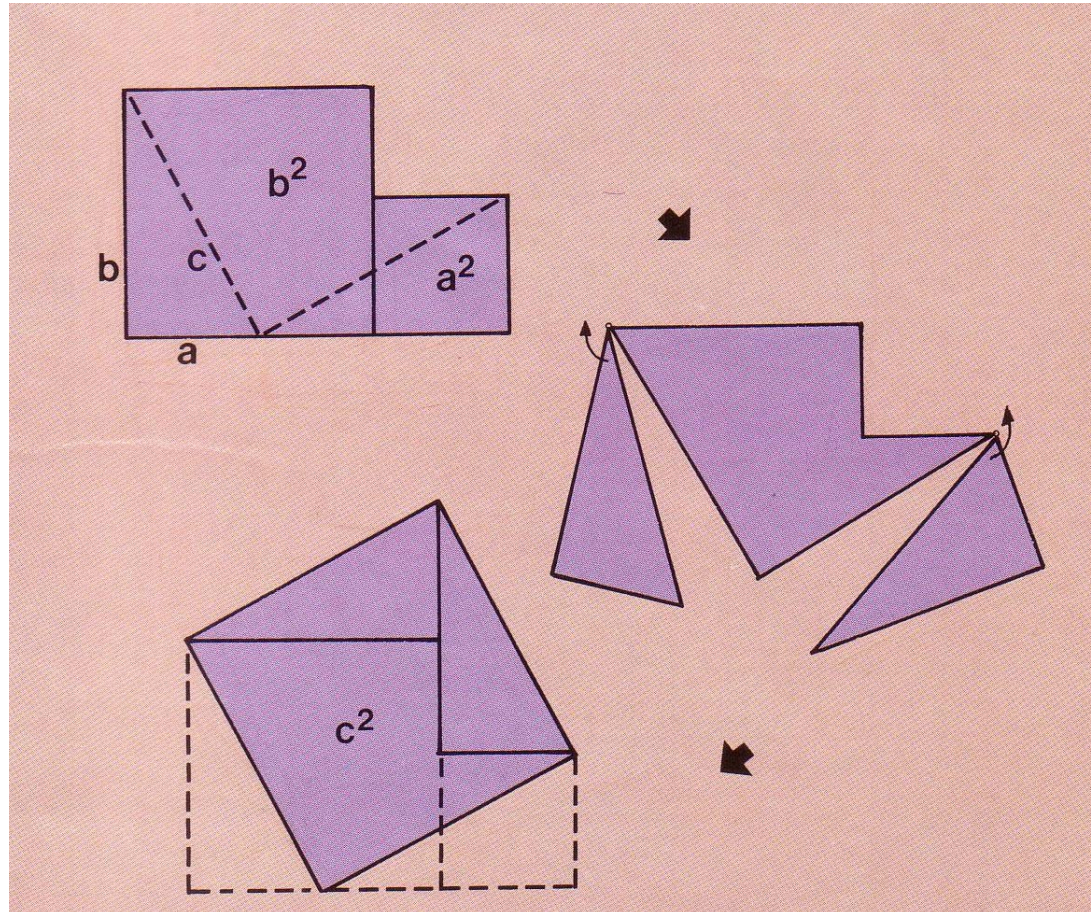
Proof of Pythagoras Theorem

Method 4

4. A figure very similar to that on the top appears as a block print in an ancient edition of the Chou Pei Suan Ching. This is often referred to as the "Chinese Proof".



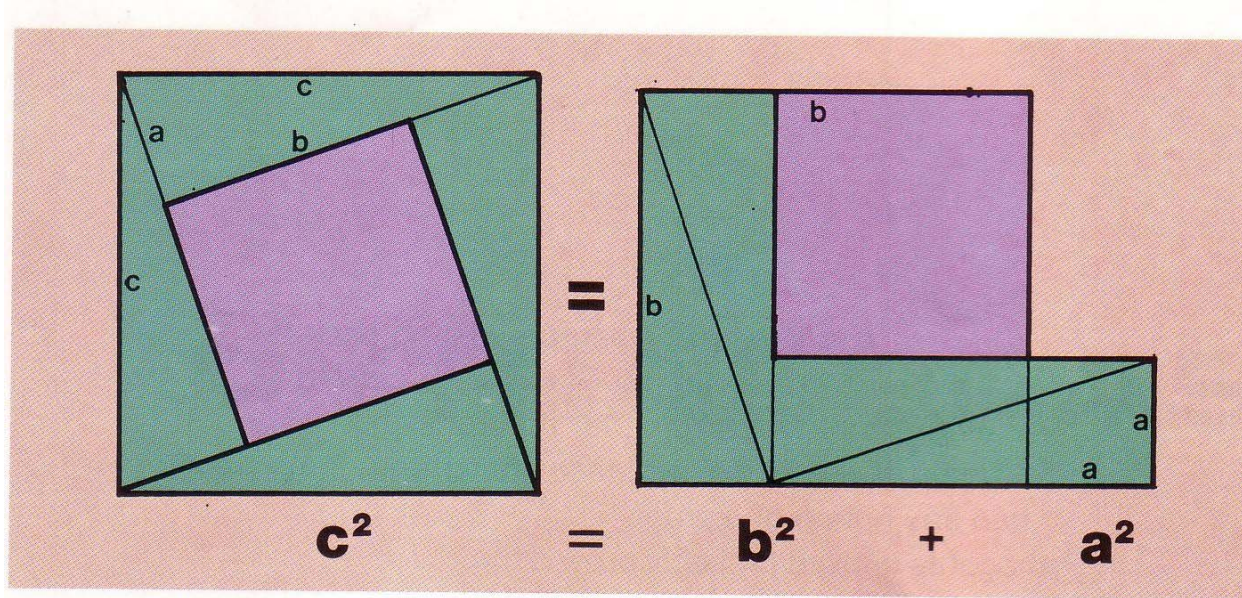
Proof of Pythagoras Theorem Method 5



5. This proof is said to have been known to Tabit ibn-Qorra, 826-901, considered to be one of the most brilliant and accomplished scholars produced by the Arabs.

Proof of Pythagoras Theorem

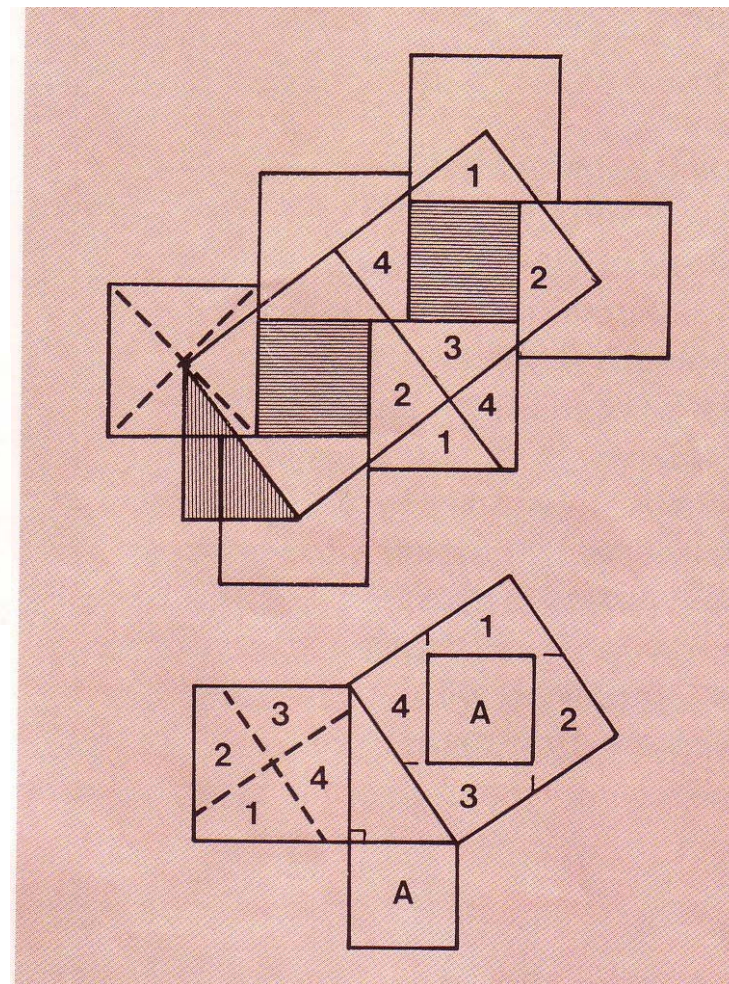
Method 6



6. The most abbreviated proof was given by the Hindu scholar Bhaskara, c. 1150, who offered the figure on the left without any explanation, only the exhortation "Behold!"

Proof of Pythagoras Theorem Method 7

7. This popular textbook proof was developed by H. Perigal in 1873.



Proof of Pythagoras Theorem

Method 8

8. Another dissection proof devised around the same time as Perigal's Dissection.

