On finding the maximum of
\[ f(x) = a \cos x + b \sin x \]

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Functions of the form \( f(x) = a \cos x + b \sin x \) occur frequently in pure and applied mathematics. Very often in school work we have to find the maximum value of \( f(x) \). The following method used by me for teaching slow learners seem to be more effective that the usual method by differentiation or by the "R \cos (x+\alpha) method".

Consider 3 rigid rods \( BC=a, CA=b, AB=\sqrt{a^2+b^2} \) forming a right angled \( \Delta \) at \( C \). The system is pivoted freely at \( B \) on a table and \( \triangle ABC \) is in a vertical plane, as shown in the figure. A light is placed above \( \triangle ABC \), and \( \Delta ABC \) is rotated anticlockwise about \( B \) with \( BA \) initially on the table. It may be seen that the shadow cast by the triangle is \( BA' \) which is of length \( a \cos x + b \sin x \) where \( x \) is the angle which \( CB \) makes with the horizontal table. The shadow is longest when \( BA \) is on the table and its length is then \( \sqrt{a^2+b^2} \) and

\[ x = \tan^{-1}\left(\frac{b}{a}\right). \]

Hence the maximum value of \( a \cos x + b \sin x \) is \( \sqrt{a^2+b^2} \) and \( x = \tan^{-1}\left(\frac{b}{a}\right) \).

It is then not difficult to convince the pupils that the least value is \( -\sqrt{a^2+b^2} \). I would like to know from teachers what visual aids could be used for the minimum value. Negative shadow?