GUIDELINES FOR DISCUSSION

Topic 1: Discovery versus rote learning in primary mathematics

1.1 What is the discovery approach in teaching mathematics? How much of primary mathematics syllabus can be learnt through discovery?

1.2 Can the discovery approach be made effective in our schools which are having an average class size of 40?

1.3 Do pupils necessarily learn best by the discovery approach?

1.4 How much importance should be given to the discovery approach in learning mathematics? What is the teacher's role in this approach? What are the advantages and disadvantages of this method?

1.5 How could the teaching and learning of mathematics be made more purposeful? How can the teaching be linked up with actual problems and practical applications? Give examples.

1.6 What is meant by rote learning? Is there a place for rote learning in mathematics?

1.7 What part should drill and mental sums play in the teaching of mathematics at the primary level? Suggest topics that can be taught effectively by this method and how the drill could be carried out.

Topic 2: Teaching of the four arithmetical operations

2.1 How can the four arithmetical operations be taught in an integrated approach?

2.2 Is memorisation of facts, e.g. multiplication tables,
desirable? Or, would you say that multiplication tables should not be memorised at all?

2.3 Is it useful for teachers to employ instructional materials in the teaching of the four arithmetical operations? Can pupils learn better through the use of instructional materials such as the mathematical balance, cuisenaire rods?

2.4 Pupils often find it difficult to solve word problems. What are the reasons? How can teachers help them in this particular aspect?

Topic 3: Problems of teaching primary mathematics in English

3.1 What are the difficulties encountered in implementing the policy of teaching mathematics in English in all schools?

3.2 Consider each difficulty listed in 3.1 and suggest solutions to overcome the difficulties.

3.3 Does language deficiency affect mathematics learning? If so, in what ways can we improve our pupils' performance in mathematics bearing in mind their language deficiency?

3.4 Pupils who learn mathematics in the second language often have no difficulty in working out problems requiring computational skills only. The main difficulty that pupils face is to comprehend word problems. Do you agree? Do pupils who learn mathematics in their first language face the same difficulty?

Topic 4: Evaluation in primary mathematics

4.1 Our teachers have been cautioned not to teach for testing. How can tests be designed to teach?

4.2 Are the skills for primary mathematics described in the Table of Specification for each primary level provided by the Ministry of Education precise and
clear enough for measurement purposes?

4.2.1 If not, what problems have been encountered in following these tables when designing a test?

4.2.2 What alternative classificatory schemes can be suggested?

4.3 What do you think of the multiple-choice objective type of questions as a means of evaluating pupils' achievement? Should it be encouraged in the school or discouraged? Give reasons and suggest alternative methods.

Topic 5: Decimals and fractions at the primary level

5.1 Comments on the primary mathematics syllabus pertaining to decimals and fractions.

5.2 Are decimals and fractions adequately taught in our primary school? How would you teach decimals and fractions?

5.3 Do teachers face any difficulty in teaching decimals and fractions? If so, what are the difficulties?

5.4 The concepts of 'the whole' and 'parts of the whole' are very abstract. What possible misconceptions could arise?

List errors commonly made by pupils in the study of decimals and fractions.
I Topic 1: Discovery versus rote learning in primary mathematics

1.1 Most subjects can be learnt through the playway method which applies also to all areas of mathematics using teaching aids supported by formal teaching.

1.2 In a class of 40, it is physically not possible for the discovery method to be effective. The only possible solution is through group work where 'discovery' can be made practicable. Each group should consist of both fast and slow learners.

1.3 Learning by the discovery approach is practicable and useful only in the initial stages of a child's education. As he advances to a higher level, he needs also to learn by rote though this must not be forced on him.

1.4 Many participants recognise the part discovery can play in learning. The teacher's role is to encourage innovations with the help of teaching aids. Children too have their part to play in bringing their own apparatus. The advantage of this method is that children learn best through their own discoveries. The disadvantages are that they tend to forget quickly what they learn, and the adoption of this method is usually at the cost of discipline.

1.5 Mathematics in class must be related to practical demands and uses in life. Participants quoted
cases of hawkers' and shopkeepers' children in their classes who are good in mathematics outside the classroom but poor in classwork. For example, these children can give change very quickly, thus showing an ability to add and subtract. Language plays an important part in the learning of mathematics. If children outside the classroom deal efficiently with mathematics in their own mother tongue then their weakness in the subject in school is due to the language problem.

1.6 Rote learning is not only committing formulae to memory, but also drilling what has been discovered and learnt. Since children forget easily, rote learning plays an important part as it helps retention.

1.7 Drills and mental sums are important and should be given prominent place. The topics that can be taught are fractions, percentages, hours and minutes. Drills can be carried out by systematic repetition of what has been taught.

II Topic 3: Problems of teaching primary mathematics in English

3.1 It is an accepted fact that English is a foreign language to most students. The switch by some students from the Chinese to the English stream adds to this inherent difficulty. The wording of the mathematical problems poses difficulty in comprehension.

3.2 Strengthen comprehension, widen vocabulary, and use simple English in mathematical problems.

3.4 Language does play an important role in comprehending mathematical problems. For students who are weak in English, language is a stumbling
block. This applies particularly to Chinese stream students having to do mathematics in English. The problem is not so serious among English medium students. On the other hand, the mathematics paper in the P.S.L.E. is worded in both Chinese and English. Therefore, this does not pose any difficulty for Chinese stream students though it has a disadvantage in that some principals tell teachers to teach mathematics in Chinese and students to answer in Chinese. Hence, when the students change streams, the language problem is enhanced. It is suggested that Chinese should not be used in the mathematics paper in the P.S.L.E.

III Topic 4: Evaluation in primary mathematics

4.1 Firstly, tests should be designed so that answers come from the children without any prompting from the teachers. Secondly, diagnostic tests should be provided to find out how much the children know. Lastly, remedial work should be given after each test to reduce weakness in the subject.

4.2 The Ministry's Table of Specification for each primary level is considered clear and precise.

4.3 Many teachers think that multiple-choice questions should be reduced and open-end questions should be increased. Multiple-choice questions encourage guess-work and are subject to chance. Distractors are unfair to students. Problem questions should be stressed as they have relevance to everyday life. Working out a sum is important as it shows how the child's mind works.
I. Topic 2: Teaching of the four arithmetical operations

2.1 At present the teaching of addition and subtraction is being integrated in the first year of school. After the child has mastered the art of addition, subtraction is introduced. With the help of aids e.g. sticks, marbles, mosaic chips etc. the child will eventually see that subtraction is the inverse of addition, e.g. \( 88 + 0 = 5 \), \( 88 - 88 = 1 \). Here number bonds can be introduced. When multiplication is taught, the rule of addition is applied first, e.g. \( 2 + 2 + 2 = 3 \) times \( 2 \). Tables are built up along this manner. When the child is taught the use of apparatus e.g. sticks, beads, etc. he gets the idea that tables are built from grouping similar numbers, e.g. \( \boxed{111} + \boxed{111} + \boxed{111} = 3 \) groups of \( 3 = 3 \times 3 = 9 \). This idea of grouping is later extended to the introduction of division e.g. put (divide) 15 sticks into groups of 3. After this informal introduction the division sign is introduced e.g. \( 15 \div 3 \). Later the child begins to relate division with multiplication, e.g. \( 2 \times 5 = 10 \), \( 10 \div 2 = 5 \), \( 10 \div 5 = 2 \). The final stage in which the four arithmetical operations can be integrated comes in the form of "number stories". Here a number e.g. 6 is given to the child. The child has to make use of the four operations to make up his own sums to arrive at an answer e.g. 6. For instance \( 12 \div 2 = 6 \), \( 3 + 3 = 6 \), \( 15 - 9 = 6 \), \( 2 \times 3 = 6 \) etc. Children love doing
this and they can come up with quite a large number of sums.

2.2 The children should first be taught that each entry in the multiplication table is obtained by repeated addition of a particular number, e.g., 3+3+3+3 = 4 times 3. When the children have understood this then they should be made to memorise their tables. By Primary 3, or at the latest, Primary 4, children should know their tables very well so that they will be able to work out mathematical problems with accuracy and speed.

2.3 It is definitely useful for teachers to employ instructional materials, especially in the teaching of concepts regarding the four operations.

2.4 Pupils of ten find it difficult to solve word problems due mainly to language difficulty. In most cases concepts are usually taught first followed by application in the form of drill problems. Word problems are usually dealt with last. However, many pupils are unable to solve these simply because they cannot comprehend what they read, not because they have not understood the concept. Teachers can help the pupils in this aspect of mathematics teaching in the following ways by giving a daily dose of simple word problems in the form of mental sums; giving more oral practice during the lesson; making pupils work out many examples of the same type of word problems they are weak in and substituting different numbers each time; helping pupils to understand the word problems and to look out for what is asked for;
teaching them to pick out key words in a word problem e.g. "left", "altogether", "product", "sums", etc.

II Topic 4: Evaluation in primary mathematics

4.1 Tests can be designed and then used as instruments in teaching. For instance pretests may find out whether children have mastered some basic knowledge of the topic to be taught so as to enable the teacher to gauge the point from where he should start to teach, and diagnostic tests may find out what pupils have not understood and may indicate to the teacher what has to be retaught.

4.2 The Tables of Specifications provided by the Ministry of Education are precise and clear for measurement purposes. However, for levels other than P6, a certain flexibility should be allowed to cater for the different abilities of the pupils. In one school the pupils may be able to cope with the recommended proportion of "remembering and understanding" and "thinking" questions, e.g. in P5 level 30% for the former and 70% for the latter. However, in another school, teachers may find that by adhering strictly to this specified proportion, the majority of the pupils will fail the tests.

4.3 Multiple choice questions should be included in assessments but kept to a minimum as in most cases evaluating of pupils' work cannot be done since no working is shown. It should be discouraged as a form of daily work unless teachers can ensure that pupils show steps in
computation to arrive at an answer. Word problems should be included in assessments to evaluate pupils' achievements.

III Topic 5: Decimals and fractions at the primary level

5.1 The syllabus pertaining to decimals and fractions up to P4 is adequate and most pupils are able to cope with the work given. However, in P5 pupils have to cope not only with five aspects of fractions as are listed in the syllabus, but also harder examples of addition and subtraction which are introduced here and the L.C.M. which is used for working out problems involving the above operations. Teachers may have to spend as long as a term on these two topics in order to teach them thoroughly. As a result teachers have to rush through the other topics of the syllabus in order to cover as much ground as possible.

5.2 Decimals and fractions can be adequately taught in the primary schools if more time is spent on these topics. However, teachers find that more often than not, they have to rush through teaching the various aspects of these topics as time has to be allocated for the other parts of the syllabus. In the first stages pupils are shown with concrete examples that a fraction is a part of a whole, e.g. \( \frac{1}{2} \) means \( \frac{1}{2} \) of 2 or 1 out of 2. After this concept has been understood, equivalent fractions and the four rules are introduced. Decimals are first introduced in the form of the decimal point as used in "money". Then the concept of tenths and hundredths is introduced with the help of the geoboard or graphboard where ten rows of
tens or one hundred squares can be made use of to illustrate that \( 0.2 = \frac{2}{10} \) (2 out of 10) or \( 0.35 = \frac{35}{100} \) (35 sqs. out of 100 sqs.) After this concept has been understood, then the four rules are taught.

5.3 The greatest difficulty teachers face when teaching fractions involves the teaching of division of fractions. Pupils find it difficult to understand why the "÷" sign is changed to a "x" sign and then the division is inverted. If no effort is made to ensure that pupils understand this by helping them to discover this fact with the use of diagrams or the number lines, then they will not invert the divisor or else invert both the fraction and its divisor. When learning decimals, pupils find it difficult to understand why the divisor has to be changed to a whole number when dividing decimals by a decimal. Another problem is the inability of pupils to see why the zero at the end of a decimal number is unnecessary, e.g. 10.070. The term "decimal fractions" should not be used as children may get confused and make the mistake of writing for e.g. 0.25½, when they divide a number. The term "decimals" should be used on its own.

5.4 On the whole pupils have no difficulty in being able to differentiate "the whole" or "parts of the whole" when these are dealt with separately. The problem arises usually in the teaching of decimals where children do not understand that the decimal point separates the whole numbers, where the smallest is the unit, and the decimal part.

5.5 The following examples show the type of errors
children usually make in the study of decimals and fractions. These pupils cannot find the common denominator, e.g. \( \frac{1}{3} + \frac{2}{3} = \frac{3}{5} \);
multiply instead of inverting the divisor, e.g. \( \frac{2}{3} \div 4 = \frac{3}{2} \times \frac{1}{4} \);
forget to change mixed numbers to improper fractions before any cancellation can be done, e.g. \\
\[
\frac{2}{5} \times \frac{9}{10} = \frac{3}{5} \\
\]
mix decimals with fractions during the process of dividing a number, e.g. \( 1.14 \div 4 = 0.2825 = 0.28\frac{1}{4} \);
fail to place decimal points or totally ignore the divisor which should be converted to a whole number first, e.g. \( 0.15 \div 0.3 = 5 \);
cancel the decimal place from left to right after a decimal number has been multiplied. e.g. \\
\[
0.12 \times 0.2 = 0.024 \\
\]
cancel off the zero at the end before placing the decimal point, e.g. \( 0.12 \times 0.5 = 0.006 \);
are unable to place the correct place value of numbers in the proper columns, e.g. \( 1+1.2 + 0.01 + 11 \) may be written as \( 1.2 + 1 + 0.01 + 11 \).
I. Topic 2: Teaching of the four arithmetical operations

2.1 At P1 and P2 levels the pupils learn the integration of two arithmetical operations on integers and at P3 level they learn the integration of the four arithmetical operations. The suggested methods in the teaching of the four arithmetical operations in an integrated approach are in the form of play, grouping, sharing and counting.

2.2 Memorization of facts like multiplication tables is desirable. However it is important that pupils must first understand the concept before they are made to memorize the facts.

2.3 It is useful for the teachers to employ instructional materials in the teaching of the four arithmetical operations. Instructional materials should only be used when introducing a new concept and should be withdrawn when pupils have understood the concept.

2.4 Pupils often find difficulty in solving word problems and the reason are they are poor in reading; they can read but cannot understand what they are reading; they can read but their reasoning power is poor; they have difficulty in expressing themselves. Teachers can help pupils solve word problems in the following ways: Give pupils problems which relate to their daily lives and the words chosen should be within their
comprehension.
More oral work should be given and mental sums should be emphasized.
Pupils should be taught to read and comprehend the problems.
Pupils should be familiar with mathematical terms.
Guided statements should be given to pupils at P1 to P3 levels.
One sentence statements should be introduced at the initial stage.

II Topic 4 : Evaluation in primary mathematics

4.1 Teachers agreed that tests should be designed for the purpose of evaluation. It was also suggested that Ministry officials give examples on how tests can be designed.

4.2 The Tables of Specifications provided by the Ministry of Education are precise and clear for measurement purposes for P1, P2, P3 and P6 levels. As whole numbers are not stated in the Tables of Specifications for P4 and P5 levels, some participants would like to know whether whole numbers should be included in the examinations. Some topics have a lot to cover but the weightage given to such topics is too little. It was suggested that the weightage for such topics should be increased.

4.3 The multiple-choice objective type of questions is not an accurate way of evaluating pupils' achievement. The multiple-choice objective type of questions should be discouraged as pupils answer the questions without thinking and are lazy to work out the sums. The alternative methods of evaluating pupils' achievement are guided problems.
III Topic 5: Decimals and fractions at the primary level

5.1 The primary mathematics syllabus pertaining to decimals and fractions is adequate. However teachers were confused by the term decimal fraction and would like a clarification on what are decimal, fraction and decimal fraction.

5.2 The following teaching aids were suggested for teaching fractions:
- concrete materials, fraction charts, geoboard,
- graphboard for fractions, graph paper, the discovery and sharing method, the practical approach.
It was also suggested that the fraction of a whole and a group should be taught side by side.

5.3 Teachers do not find difficulty in the teaching of addition and subtraction of fractions but encounter difficulties in the teaching of multiplication and division of fractions. It is difficult to put across to the pupils the concept that when a fraction is multiplied by another fraction, the answer may be smaller in value, but when a whole number is multiplied by another whole number, the answer is larger in value. The same applies to the division of fraction by a fraction. It is also difficult to make pupils understand equivalent fractions.

5.4 The possible misconceptions that could arise when teaching the concepts of 'the whole' and 'parts of the whole' are that pupils may not visualise the group as a whole, they may visualise part of an apple as a whole and not as a fraction of a whole and they may not understand that a group is also a whole.

5.5 The following are errors commonly made by pupils in the study of fractions:
When doing addition of fractions, pupils add the numerators and the denominators separately, e.g. $\frac{1}{3} + \frac{1}{4} = \frac{2}{6}$. When doing addition of fractions, where the L.C.M. is not required, e.g. $\frac{1}{3} + \frac{1}{4}$, pupils are not sure whether to reduce 8 to 4 or to increase 4 to 8.

When reducing a fraction to its lowest term, pupils do not make use of the common factor.

When doing multiplication of fractions, pupils forget to change mixed numbers into improper fractions.

When doing addition or subtraction of fractions involving mixed numbers, the whole numbers are always left out in the final answers.

When doing multiplication of fractions involving mixed numbers, pupils work on the fractions first and at the final stage they simply put down the whole numbers.

When doing division of a fraction by a fraction, instead of inverting the second fraction pupils invert the first fraction.

When doing multiplication of fractions involving mixed numbers, pupils add the whole numbers to the numerator. Pupils also multiply the whole number by the whole number and the fraction by the fraction.

The following are errors commonly made by pupils in the study of decimals:

Wrong placing of the decimal point.

Forgetting to put the decimal point.

When doing multiplication of decimals, pupils forget to count the number of decimal places.

The pupils think the longer decimal should have the greater value, e.g. 2.145 is greater in value than 21.

When doing division of decimals, the zero in the
decimal is left out, e.g. 0.048 ÷ 8 = 0.6 and 0.808 ÷ 8 = 0.11.

When doing division of decimals a fraction is often placed after the decimal, e.g. 10.5 ÷ 2 = 5.2½. 
I  Topic 1: Discovery versus rote learning in primary mathematics

1.1 The discovery approach is incidental learning. The teacher creates the environment for the children to discover a concept by themselves. The teacher must guide the children in all the activities so there is no pure discovery approach. All the topics especially those involving formulae can be learnt through discovery.

1.2 The discovery approach is effective only with the brighter children. Even with the teacher's guidance, the slow learners find it difficult to discover the concept by this method. The teacher has to use the chalk and talk method.

1.3 Yes.

1.4 Teachers will give the discovery method as much importance as time allows. Their role is to motivate and supervise. The advantages of the method are: children understand the concept and will not easily forget it; they enjoy learning and learning becomes meaningful. The disadvantages are: time factor, the size of the class, the teacher's limited mathematical knowledge and the teacher's lack of confidence and ingenuity.

1.5 Teaching and learning could be made more meaningful by the use of teaching aids and by getting the children to work on projects. The examples given by the teacher should relate to the children's day to day experiences. Examples:
(i) on the value of money - the teacher could refer to the price lists of consumer goods advertised by the various supermarkets; (ii) on areas - get the children to measure the floor and find the cost of tiling it; (iii) on simple interest - teach the topic in relation to the children's POSB savings; (iv) on graphs - teach in connection with POSB savings, and the number of children who drink milk in each class.

1.6 Rote learning means committing facts to memory. There is a place for rote learning, e.g. spelling of numerals, multiplication tables, conversion tables and formulae.

1.7 Drill and mental sums should be given as a warming up before the start of a lesson. Topics like multiplication tables, measurement, area and volume can be drilled after the children have understood the concepts by the discovery approach.

II Topic 2: Teaching of the four arithmetical operations

2.1 The integrated approach means that the teaching of the four operations should not be compartmentalized. All the teachers agreed that the four operations should be introduced separately. When the children have mastered the skills the teacher could integrate the rules, e.g. teach addition in relation to multiplication.

2.2 Facts like multiplication tables should be memorised once the children have got the concepts.

2.3 It is always better to use teaching aids to teach the four arithmetical operations, but when the children have mastered the skills, these aids should be withdrawn.

2.4 The main reason is language deficiency. The
children cannot comprehend the problem. Some children develop "a mental block" the moment they come across a problem because they cannot write statements. They refuse to try the sum. Key words like "altogether" and "how much more" should be used to help the children solve the problems. Teachers should encourage but should not insist on the children writing statements, for one of the aims in mathematics is to teach computational skills and not language skills. In an examination, no marks should be deducted for not writing statements. A teacher suggested that children should be taught to read the problem slowly and pause at every phrase.

III Topic 4: Evaluation in primary mathematics

Those present discussed how often the tests should be carried out. Many agreed that there should be three assessments in a semester. All the children from the same standard should sit for a common test. The test should be graded to cater for the bright, average and slow learners. The setter should refer to the table of specifications when setting the test. As for the slow learners the teacher need only to teach the basic skills.
GROUP E

Chairman : Mr. Peter Goh Lye Heng
Rapporteur : Mrs. John Sng

I Topic I: Discovery versus rote learning in primary mathematics

1.1 What is to be learnt is not to be presented directly through the teacher. The discovery approach means that both reception learning and discovery learning can be very meaningful. They are also inter-related. Therefore it is impossible to measure how much of the primary mathematics syllabus can be learnt through discovery.

1.2 The discovery approach may not be effective in large classes at P1 and P2 levels, especially with children under 9 years of age. It is effective to a certain extent at P4 level and above.

1.3 Pupils do not necessarily learn best by the discovery approach.

1.4 Not too much time should be given to the discovery approach. It can be made use of only when we teach the concepts of a topic. The teacher should create a situation to provide opportunities for children to learn through discovery. The teacher should be mathematically sensitive enough to help children out when a problem arises. The advantages of this method are that learning becomes more meaningful and what is learnt has to some extent a more lasting effect in the children's minds. The disadvantages are that the method is time-consuming and suitable only for small classes, and that the teacher must have a good knowledge of his pupils' abilities. Children under 9 years
of age tend to be impulsive and jump to conclusion, so guidance is more important.

1.5 The role of the teacher is of the utmost importance. He must be deeply involved in providing situational teaching and experiences relevant to the topic taught. For example, when he teaches "money and weights", the market and supermarket scene should be created. Children are to be encouraged in taking an active part in such situations. When he teaches "fractions", children can take part in the cutting of a cake into $\frac{1}{4}$, $\frac{1}{8}$ or $\frac{1}{12}$.

1.6 Rote learning means learning without meaning and understanding. It has no place in mathematics.

1.7 Drill is used to help children to remember better. Mental sums help to stimulate their thinking power, to exercise their minds and to increase speed in their work. Drill can be carried out through constant daily repetition, say for five or ten minutes. They should be drilled on the multiplication tables, the four operations and formulae. The practice of doing mental sums should also be re-introduced in the schools.

II  Topic 5: Decimals and fractions at the primary level

5.1 The syllabus for the teaching of decimals and fractions is fairly adequate. However, more time is needed to cover the two topics if we want to make sure that the children understand them well and are able to retain what they have understood.

5.2 Decimals and fractions are quite adequately taught in our primary schools. Illustrations can be used to introduce fractions. For example, $\frac{1}{4}$ or $\frac{1}{8}$ can be taught by putting up drawings, such as the cutting of a cake or an orange, and
the folding of pieces of paper, etc. It is important to teach the concept well. Once the children have mastered the concept well then decimal fractions can be introduced to them. From the concept of a fraction the notion of a decimal fraction is then introduced. For example, show the children that $\frac{2}{10}$ is 2 parts out of ten parts and the decimal fraction 0.2 also represents 2 parts out of ten parts, so, instead of writing $\frac{2}{10}$, we can also write it as 0.2.

5.3 Quite often difficulty is found at P4 level. The children tend to lose interest as they are unable to retain so many topics that are crammed into their heads. This is perhaps due to a lack of understanding at the fundamental stage. Lack of time to master all that have been taught is also an important factor contributing to the difficulty. Very often the concept is not well taught. Children also find difficulty in dividing a fraction by another fraction, e.g. $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$. How can the teacher teach this concept in a simple way?

5.4 The concepts of 'the whole' and 'parts of the whole' are very abstract. Therefore, in the teaching of fractions we must show children that 'the whole' can be taken in terms of one unit or as a group. Example:

- 1 unit
- 1 group

5.5 Example: Which is greater, $\frac{1}{6}$ or $\frac{1}{5}$? Children often say that $\frac{1}{6}$ is greater than $\frac{1}{5}$. They tend to compare the denominators and do not look at the fractions as a whole.