FINITE 2-GROUPS OF CLASS TWO WITH CYCLIC CENTRE*

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It was discovered by J.M. Brady (Bull. Austral. Math. Soc. 1 (1969), 403-416) that a finite $q$-group of nilpotency class two with cyclic centre is a central product either of two-generator subgroups with cyclic centre or of two-generator subgroups with cyclic centre and a cyclic subgroup, and that the finite $q$-groups of class two on two generators with cyclic centre comprise the following list:

- $2r \leq n$: $Q(n,r) = \langle a, b : a^{q^n} = b^{q^r} = 1, a^{q^{n-r}} = [a, b] \rangle$;
- $r < n < 2r$: $Q(n,r) = \langle a, b : a^{q^n} = b^{q^r} = 1, a^{q^r} = [a, b]^{q^{2r-n}} \rangle$;

$[[a, b], a] = [[a, b], b] = 1$;

and if $q = 2$ we have as well

- $n > 1$: $R(n) = \langle a, b : a^{2^{n+1}} = b^{2^n + 1} = 1, a^{2^n} = [a, b]^{2^{n-1}} = b^{2^n}, [[a, b], a] = [[a, b], b] = 1 \rangle$.

In conjunction with an earlier work (J. Austral. Math. Soc. 17 (1974), 142-153) which deals with the odd-order case, this paper completes the solution of the isomorphism problem for finite nilpotent groups of class two with cyclic centre by obtaining a canonical decomposition for 2-groups of such type and proving its uniqueness.

The main result is the following theorem.

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THEOREM. Every finite 2-group \( G \) of class two with cyclic centre, either has the central decomposition

\[
G \cong Q(n_1, r_1) \cdots Q(n_\alpha, r_\alpha) Q(l_1, 1) \cdots Q(l_\beta, 1),
\]

where \( \alpha > 0, \epsilon_i > 0, i = 1, \ldots, \beta \).

or else \( G \) has the central decomposition

\[
G \cong R(n, 0) Q(l_1, 1) \cdots Q(l_\beta, 1),
\]

where \( n > l > 1, \epsilon_i > 0, i = 1, \ldots, \beta \).

The details of the paper will appear in the *Journal of the Australian Mathematical Society*. The above decomposition is unique up to isomorphism.

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