FINITE 2-GROUPS OF CLASS TWO WITH CYCLIC CENTRE*

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It was discovered by J.M. Brady (*Bull. Austral. Math. Soc.* 1 (1969), 403-416) that a finite q-group of nilpotency class two with cyclic centre is a central product either of two-generator subgroups with cyclic centre or of two-generator subgroups with cyclic centre and a cyclic subgroup, and that the finite q-groups of class two on two generators with cyclic centre comprise the following list:

$$2r \leqslant n: \quad Q(n,r) = \langle a,b : a^{q^n} = b^{q^r} = 1, \quad a^{q^{n-r}} = [a,b] \rangle;$$

$$r \leqslant n < 2r: \quad Q(n,r) = \langle a,b : a^{q^n} = b^{q^r} = 1, \quad a^{q^r} = [a,b]^{q^{2r-n}},$$
$$[[a,b],a] = [[a,b],b] = 1 \rangle;$$

and if q = 2 we have as well

$$n \geqslant 1: \quad R(n) = \langle a,b : a^{2^{n+1}} = b^{2^{n+1}} = 1, \; a^{2^n} = [a,b]^{2^{n-1}} = b^{2^n},$$
$$[[a,b],a] = [[a,b],b] = 1 \rangle.$$

In conjunction with an earlier work (J. Austral. Math. Soc. 17 (1974), 142-153) which deals with the odd-order case, this paper completes the solution of the isomorphism problem for finite nilpotent groups of class two with cyclic centre by obtaining a canonical decomposition for 2-groups of such type and proving its uniqueness.

The main result is the following theorem.

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THEOREM. *Every finite 2-group G of class two with cyclic centre either has the central decomposition*

$$G \cong Q(n_1, r_1) \dots Q(n_\alpha, r_\alpha) Q(l,l)^{\epsilon_1} \dots Q(1,1)^{\epsilon_1},$$

where $\alpha \geqslant 0$, $\epsilon_i \geqslant 0$, $i = 1, \dots, l$.

$$n_1 > \dots > n_\alpha > l \geqslant 1, \qquad n_\alpha > r_1 > \dots > r_\alpha \geqslant 0,$$

$$1 < n_1 - r_1 < \dots < n_\alpha - r_\alpha,$$

and $Q(n,0)$ *is the cyclic group of order* 2^n;

or else G has the central decomposition

$$G \cong R(n) Q(l,l)^{\epsilon_l} \dots Q(1,1)^{\epsilon_1},$$

where $n \geqslant l \geqslant 1$, $\epsilon_i \geqslant 0$, $i = 1, \dots, l$.

The above decomposition is unique up to isomorphism.

The details of the paper will appear in the *Journal of the Australian Mathematical Society*.