SOME RESULTS ON METANILPOTENT GROUPS*

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Let G be a group. Let $G = \Gamma_1(G) \geqslant \Gamma_2(G) \geqslant \ldots$ be the lower central series of G with $\Gamma(G)$ as its terminal member. A finite group G is said to be *metanilpotent* if and only if $\Gamma(G)$ is nilpotent. In this talk we extend to metanilpotent groups some results of Huppert [1] and Inagaki [2] on groups with nilpotent commutator subgroups.

The first two results are generalizations of theorems of Huppert [1, Satz 4 and Satz 5].

THEOREM 1. *Let G be a finite group such that $\Gamma_i(G)$ is nilpotent for some i. Then $\Gamma'_i(H)$ is normal in G for every Hall subgroup H of G.*

THEOREM 2. *Let G be a finite group. If G is metanilpotent, then the Sylow p-subgroup of the Frattini subgroup $\Phi(G)$ is the intersection of the Frattini subgroups of all the Sylow p-subgroups of G.*

The third result is a generalization of a theorem of Inagaki [2, Theorem 3].

THEOREM 3. *Let G be a finite group. Let N be a normal subgroup of G. Suppose that there exists a maximal subgroup M of G such that $M \cap N \leqslant \Phi(G)$. Then the following results hold:*

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(1) *N is nilpotent.*

(2) *The index of M in G is a power of a prime p.*

(3) *If P is a Sylow p-subgroup of G, where p is the prime in (2), then P is normal in G and G/P is nilpotent.*

REFERENCES

1. B. Huppert, *Normalteiler und maximalen Untergruppen endlicher Gruppen, Math. Z.* 60 (1954), 409-434.

2. N. Inagak *On groups with nilpotent commutator subgroups, Nagoya Math., J.* 25 (1965), 205-210.