

## PROBLEMS AND SOLUTIONS

Members are invited to contribute to the problems as well as the solutions of our Problems and Solutions section. Interesting problems at secondary school level are most welcome. A book-voucher prize will be awarded to the best solution of a starred problem sent in by Junior Members. If equally good solutions are received, the prize will be awarded to the solution sent with the earliest postmark. In the case of identical postmarks, the winning solution will be decided by ballot.

Problems or solutions should be sent to Dr. K.N. Cheng, Department of Mathematics, National University of Singapore, Bukit Timah Road, Singapore 1025.

P1/80 Find all real  $x$  which satisfy the equation

$$\sqrt{x - 2\sqrt{x-1}} + \sqrt{x+3 - 4\sqrt{x-1}} = 1.$$

(via Teo Soh Wah)

P2\*/80 Without obtaining its value, show that the integral  $\int_0^{\infty} \cos(x^2) dx$  is positive.

(via K.M. Chan)

P3/80. Simplify, for  $x > 1$ , each of the following:

(i)  $\sqrt{\frac{x-1}{x+1}} + \frac{2}{(x+1) + \sqrt{x^2-1}}$

(ii)  $\arcsin \frac{x+a+\sqrt{x^2-1}}{\sqrt{1-a^2}} - \arcsin \sqrt{\frac{(1-a)(x-1)}{(1+a)(x+1)}}$ ,  $|a| < 1$ .

(iii)  $\left[ \frac{x+a+\sqrt{x^2-1}-\sqrt{a^2-1}}{x+a+\sqrt{x^2-1}+\sqrt{a^2-1}} \right] \left[ \frac{\sqrt{(a+1)(x+1)}-\sqrt{(a-1)(x-1)}}{\sqrt{(a+1)(x+1)}+\sqrt{(a-1)(x-1)}} \right]$ ,  $a > 1$ ,

(iv)  $\left[ \frac{x+a+\sqrt{x^2-1}-\sqrt{a^2-1}}{x+a+\sqrt{x^2-1}+\sqrt{a^2-1}} \right] \left[ \frac{\sqrt{(a^2-1)(x^2-1)}-(ax+1)}{x+a} \right]$ ,  $a > 1$ .

(via M.J. Wicks)