SINGAPORE MATHEMATICAL SOCIETY

Interschool Mathematical Competition 1987

Part A

Saturday, 27 June 1987 1000–1100

Attempt as many questions as you can.
No calculators are allowed.
Circle your answers on the Answer Sheet provided.
Each question carries 5 marks.

1. Let $x = (log_2 3)(log_3 4)(log_4 5)(log_5 6)(log_6 7)(log_7 8)$. Then

(a) $1 < x < 2$ (b) $2 < x < 3$ (c) $3 < x < 4$ (d) $4 < x < 5$

(e) None of the above.

2. Consider the following truncated chessboard with 8 squares. We say that two squares are *touching* if they have at least one common vertex. In how many ways can you assign the numbers 1,2,3,4,5,6,7,8 to the squares in such a way that different squares are assigned different numbers and the numbers assigned to two touching squares are not consecutive?

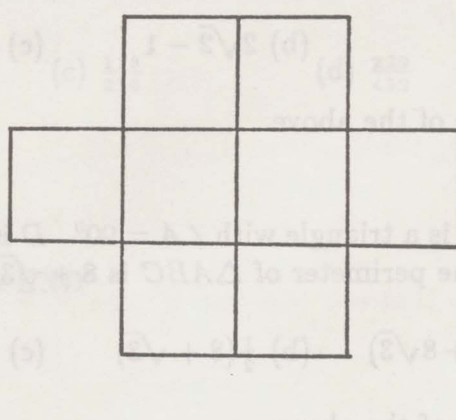
(a) 0

(b) 2

(c) 4

(d) 8

(e) None of the above.



3. The sum of the squares of all real numbers satisfying the equation $x^{256} - 256^{32} = 0$ is

(a) 2 (b) 4 (c) 6 (d) 8

(e) None of the above.

4. The value of $\frac{2\times 1}{2^1} + \frac{2\times 2}{2^2} + \frac{2\times 3}{2^3} + \ldots + \frac{2n}{2^n} + \ldots$ is

(a) 3.7 (b) 3.9 (c) 4.1 (d) 4.2

(e) None of the above.

5. Let $x = \sqrt{5+\sqrt{3+\sqrt{5+\sqrt{3+\ldots}}}}$. Then

(a) $2 < x < 3$ (b) 3 (c) $3 < x < 4$ (d) 4

(e) None of the above.

6. The value of

$$\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$$

is

(a) 1 (b) $2\sqrt{2}-1$ (c) $\sqrt{5}/2$ (d) $\sqrt{5/2}$

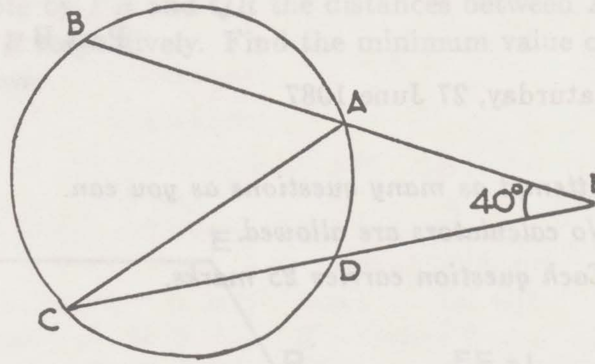
(e) None of the above.

7. ABC is a triangle with $\angle A = 90^0$. D is the midpoint of BC and $AD = 2$. If the perimeter of $\triangle ABC$ is $8+\sqrt{3}$, then the area of $\triangle ABC$ is

(a) $\frac{1}{4}(3+8\sqrt{3})$ (b) $\frac{1}{2}(8+\sqrt{3})$ (c) $3\sqrt{3}$ (d) 4

(e) None of the above.

8. In the following figure, arc AB, arc BC and arc CD are of equal lengths. The angle ACD is



(a) 10^0

(b) 15^0

(c) 20^0

(d) 25^0

(e) None of the above.

9. Six persons are randomly chosen from seven couples. The probability that there are exactly two couples among the six chosen is

(a) $\frac{10}{143}$ (b) $\frac{20}{143}$ (c) $\frac{40}{143}$ (d) $\frac{45}{143}$

(e) None of the above.

10. Amy and her brother are playing chess. Amy's chances of winning and losing a game are $\frac{1}{2}$ and $\frac{1}{3}$ respectively; and the chance that a game ends in a draw is $\frac{1}{6}$. The winner of a game gets 1 point, and the loser gets 0 point; if they draw, each gets $\frac{1}{2}$ point. After four games, what is Amy's chance of having a higher total? You may assume that the results of the games are independent.

(a) $\frac{1}{2}$ (b) $\frac{15}{32}$ (c) $\frac{113}{216}$ (d) $\frac{229}{432}$

(e) None of the above.

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Part B

Saturday, 27 June 1987 1100–1300

Attempt as many questions as you can.
No calculators are allowed.
Each question carries 25 marks.

1. Prove or disprove: There exist prime numbers a, b, c, d such that $a < b < c < d$ and

$$\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}.$$

2. Let $f(x) = a_0 + a_1 x + \ldots + a_n x^n$ be a polynomial of degree n with integer coefficients. If a_0, a_n and $f(1)$ are odd, prove that $f(x) = 0$ has no rational roots.

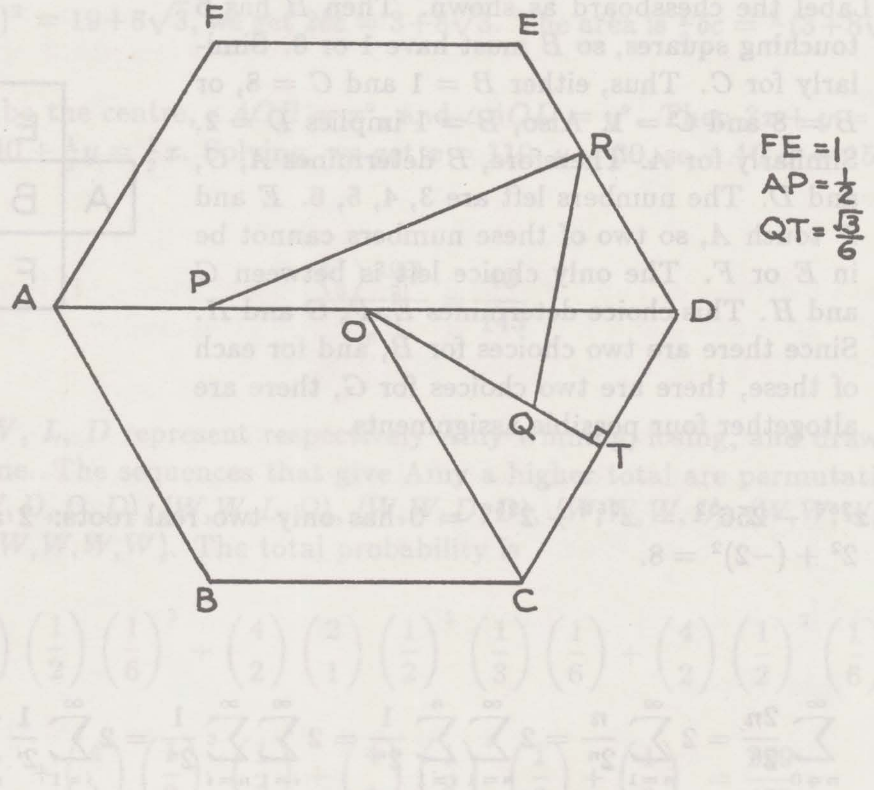
3. Four persons take their seats randomly at a circular table with nine chairs. What is the probability that no two persons sit immediately next to each other?

4. For any three angles A, B, C with

$$\cos A + \cos B + \cos C = \sin A + \sin B + \sin C = 0,$$

prove that $\cos^2 A + \cos^2 B + \cos^2 C$ is a constant. What is this constant?

5. In the following figure, $ABCDEF$ is a regular hexagon with centre 0, P is the mid-point of OA, Q is the centroid of $\triangle OCD$, and R is a variable point on the hexagon. Denote by PR and QR the distances between P and R and between Q and R respectively. Find the minimum value of $PR + QR$. Justify your answer.



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