1. \[ x = \frac{\log_2 3 \log_2 4}{\log_2 2 \log_2 3} \ldots \frac{\log_2 8}{\log_2 2} = \frac{\log_2 8}{\log_2 2} = 3. \]

2. Label the chessboard as shown. Then \( B \) has 6 touching squares, so \( B \) must have 1 or 8. Similarly for \( C \). Thus, either \( B = 1 \) and \( C = 8 \), or \( B = 8 \) and \( C = 1 \). Also, \( B = 1 \) implies \( D = 2 \). Similarly for \( A \). Therefore, \( B \) determines \( A \), \( C \), and \( D \). The numbers left are 3, 4, 5, 6. \( E \) and \( F \) touch \( A \), so two of these numbers cannot be in \( E \) or \( F \). The only choice left is between \( G \) and \( H \). This choice determines \( E \), \( F \), \( G \) and \( H \). Since there are two choices for \( B \), and for each of these, there are two choices for \( G \), there are altogether four possible assignments.

3. \( x^{256} - 256^{32} = x^{256} - 2^{256} = 0 \) has only two real roots: 2 and -2, so \( 2^2 + (-2)^2 = 8 \).

4. 
\[
\sum_{n=0}^{\infty} \frac{2^n}{2^n} = 2 \sum_{n=1}^{\infty} \frac{n}{2^n} = 2 \sum_{i=1}^{\infty} \frac{1}{2^n} = 2 \sum_{i=1}^{\infty} \frac{1}{2^i} \sum_{n=i}^{\infty} \frac{1}{2^n} = 2 \sum_{i=1}^{\infty} \frac{1}{2^i} \sum_{n=i}^{\infty} \frac{1}{2^n} = 2 \sum_{i=1}^{\infty} \frac{1}{2^i} \frac{1}{1 - \frac{1}{2}} = 4 \sum_{i=1}^{\infty} \frac{1}{2^i} = 4.
\]

5. \( x = \sqrt{5} + \sqrt{3} + x \Rightarrow x^2 - 5 = \sqrt{3} + x \Rightarrow x^4 - 10x^2 - x + 22 = 0 \Rightarrow (x+2)(x^3 - 2x^2 - 6x + 11) = 0 \). Let \( f(x) = x^3 - 2x^2 - 6x + 11 \). Then, \( f(x) < 0 \) for large negative \( x \), \( f(0) > 0 \), \( f(2) < 0 \), \( f(3) > 0 \); these imply there are 3 real roots, two of which are less than 2. But \( x > \sqrt{5} > 2 \), so \( 2 < x < 3 \).
6. \( \left( \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} \right)^2 = 2 \), and \( 3 - 2\sqrt{2} = (\sqrt{2} - 1)^2 \),
so we have \( \sqrt{2} - (\sqrt{2} - 1) = 1 \).

7. Let \( b \) be the length of \( AC \) and \( c \) the length of \( AB \). Then \( b + c = 8 + \sqrt{3} - BC = 8 + \sqrt{3} - 4 = 4 + \sqrt{3} \) and \( b^2 + c^2 = BC^2 = 16 \). Since \( (b+c)^2 = 19 + 8\sqrt{3} \), we get \( 2bc = 3 + 8\sqrt{3} \). The area is \( \frac{1}{2}bc = \frac{1}{4}(3 + 8\sqrt{3}) \).

8. Let \( O \) be the centre, \( \angle AOB = x^\circ \), and \( \angle AOD = y^\circ \). Then \( 3x + y = 360 \) and \( 40 + \frac{1}{2}y = \frac{1}{2}x \). Solving, we get \( x = 110, y = 30 \), so \( \angle ACD = 15^\circ \).

9. \[ \left( \begin{array}{c} 7 \\ 2 \end{array} \right) \frac{10 \cdot 8}{2!} \cdot \frac{40}{143} \]

10. Let \( W, L, D \) represent respectively Amy winning, losing, and drawing a game. The sequences that give Amy a higher total are permutations of \( (W,D,D,D), (W,W,L,D), (W,W,D,D), (W,W,W,L), (W,W,W,D) \) and \( (W,W,W,W) \). The total probability is

\[ \left( \begin{array}{c} 4 \\ 1 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \end{array} \right) \left( \begin{array}{c} 1 \\ 6 \end{array} \right)^3 + \left( \begin{array}{c} 4 \\ 2 \end{array} \right) \left( \begin{array}{c} 2 \\ 1 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \end{array} \right) \left( \begin{array}{c} 1 \\ 3 \end{array} \right) \left( \begin{array}{c} 1 \\ 6 \end{array} \right) + \left( \begin{array}{c} 4 \\ 2 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \end{array} \right) \left( \begin{array}{c} 1 \\ 6 \end{array} \right)^2 \\
\left( \begin{array}{c} 4 \\ 1 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \end{array} \right) \left( \begin{array}{c} 1 \\ 3 \end{array} \right) + \left( \begin{array}{c} 4 \\ 1 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \end{array} \right) \left( \begin{array}{c} 1 \\ 6 \end{array} \right) + \left( \begin{array}{c} 1 \\ 2 \end{array} \right)^4 = \frac{229}{432} \]
1. Suppose the claim is true.

\[
\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d} \Rightarrow cd(b - a) = ab(d - c),
\]

so b divides cd(b - a). Since b is a prime, either b divides cd or b divides b - a. But b, c, and d are unequal primes, so b does not divide cd; also, b ≠ a, so b does not divide b - a. These contradict the previous statement, so the claim is false.

2. Suppose \( \frac{p}{q} \) is a rational root, where p, q are integers and \( \gcd(p, q) = 1 \). Then

\[
a_0 q^n + a_1 q^{n-1} p + \cdots + a_n p^n = 0. \tag{1}
\]

Hence p divides \( a_0 \) and q divides \( a_n \), so both p and q are odd. It follows that \( a_k q^{n-k} p^k \) is odd if and only if \( a_k \) is odd. LHS of (1) is even, so \( a_0 + a_1 + \cdots + a_n \) is even. This contradicts the fact that \( f(1) \) is odd.

3. There are 8.7.6 ways to sit in a circle. The first person can sit anywhere. The other 3 permute themselves clockwise in 6 ways, with one chair between every two persons. The extra chair can go in 4 possible positions, so the total number of ways is 4.6 = 24. The probability is \( \frac{24}{8.7.6} = \frac{1}{14} \).

4. We have \( (\cos A + \cos B + \cos C)^2 = 0 \), so

\[
\cos^2 A + \cos^2 B + \cos^2 C + 2(\cos A \cos B + \cos B \cos C + \cos C \cos A) = 0. \tag{2}
\]

However,

\[
2(\cos A \cos B + \cos B \cos C + \cos C \cos A) = \cos(A + B) + \cos(A - B) + \cos(B + C) + \cos(B - C) + \cos(C + A) + \cos(C - A). \tag{3}
\]

Let \( A + B + C = D \). Then

\[
\cos(A + B) + \cos(B + C) + \cos(C + A) = \cos(D - C) + \cos(D - A) + \cos(D - B) = \cos D(\cos C + \cos A + \cos B) + \sin D(\sin C + \sin A + \sin B) = 0.
\]
Moreover, \((\sin A + \sin B)^2 = (-\sin C)^2 = \sin^2 C\) and \((\cos A + \cos B)^2 = (-\cos C)^2 = \cos^2 C\), from which we get \(2 + 2\cos(A - B) = 1\), i.e., \(\cos(A - B) = -\frac{1}{2}\). Similarly, \(\cos(B - C) = \cos(C - A) = -\frac{1}{2}\). Combining (2), (3) and these, we get \(\cos^2 A + \cos^2 B + \cos^2 C = \frac{3}{2}\).

5. It is not difficult to see that the position of \(R\) which yields minimum \(PR + QR\) must be on \(AB\), \(BC\) or \(CD\).

- **\(R\) on \(AB\).**
  \[PR + RQ = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{3}\right)^2} = \sqrt{\frac{37}{12}}\]

- **\(R\) on \(BC\).**
  \[PR + RQ = \sqrt{1^2 + \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3}\right)^2} = \sqrt{\frac{37}{12}}\]

- **\(R\) on \(CD\).**
  \[PR + QR = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}\right)^2} = \sqrt{\frac{31}{12}}\]

The minimum value is \(\sqrt{\frac{31}{12}}\).