35th International Mathematical Olympiad

Singapore participated in the 35th International Mathematical Olympiad in Hong Kong from 8 to 20 July 1994. The Singapore team consisted of Dr Wong Yan Loi (Team Leader), Miss Ong Bee Suan (Deputy Leader) and the following six students: Chor Han Ping (Raffles J C), Ho Chui Ping (Raffles J C), Pang Siu Taur (Raffles J C), Tan Choon Siang (Raffles J C), Tan Swee Heng (Raffles J C) and Wee Hoe Teck (Hwa Chong J C).

Pang Siu Taur and Wee Hoe Teck each was awarded a silver medal while Chor Han Ping, Ho Chui Ping and Tan Swee Heng each received an honourable mention. Unofficially, Singapore ranked 29 among 69 participating countries.

The Singapore mathematical Olympiad Training Committee 1994 consisted of Dr Tay Tiong Seng (Chairman), Dr Wong Yan Loi (Vice-Chairman), Mrs Chang Swee Tong, Dr Chan Onn, Assoc Prof Chen Chuan Chong, Dr Chua Seng Kee, Dr Chua Seng Kiat, Mr Hang Kim Hoo, Ms Kan Sou Tin, Assoc Prof Koh Khee Meng, Dr Leong Yu Kiang, Dr Denny Leung, Ms Lim Bee Hiong, Ms Ng Bee Huay, Ms Ong Bee Suan, Dr Roger Poh, Dr Qu Ruibin, Ms Soh Wai Lan, Dr Sun Yeneng, Mr Tai Thiam Hoo, Ms Tay Lai Ling, Dr Tay Yong Chiang.

IMO '94 Problem 6

Show that there exists a set A of positive integers with the following property: For any infinite set S of primes, there exist positive integers $m \in A$ and $n \notin A$, each of which is a product of k distinct elements of S for some $k \geq 2$.

Solution

Denote the primes by $p_2, p_3, \ldots$ such that $p_2 < p_3 < p_4 < \ldots$. For $k \geq 2$, define A_k to be the set of all numbers that are product of k distinct primes $p_{i_1}, p_{i_2}, \ldots, p_{i_k}$, where $i_1 \equiv i_2 \equiv i_3 \equiv \ldots \equiv i_k \pmod{p_k}$.

> We write $n \equiv m$ mod p if p divides $(n - m)$,
> in other words, n and m have the same remainder when divided by p.

For example $A_2 = \{p_2 p_4, p_2 p_6, p_4 p_6, \ldots\}$.

Let $A = U_{k=2}^{\infty} A_k$. We shall prove that A has the required property.

For a given infinite set $S = \{p_{s_1}, p_{s_2}, p_{s_3}, \ldots\}$ of primes,

let $S^* = \{S_1, S_2, S_3, \ldots\}$ be the set of indices. Obviously, there exists an infinite subset Q^* of S^* such that all members of Q^* are equal modulo p_k (by the pigeonhole principle). Take any k members $q_1, q_2, \ldots, q_k$ of Q^*. Then the product $p_{q_1} p_{q_2} p_{q_3} \ldots p_{q_k}$ will both be a member of A and a product of k distinct elements of S. This is true for all $k \geq 2$.

Now let the value of p_k be equal to the smallest prime not dividing $S_2 - S_1$. Hence $S_1 \not\equiv S_2 \pmod{p_k}$. Then we simply take $p_{S_1} p_{S_2} p_{S_3} \ldots p_{S_k}$. This is clearly a product of k distinct elements of S, yet not a member of A.

Pang Siu Taur was a student of Raffles Institution, and later Raffles Junior College. During that time he cultivated an interest in mathematics, and also participated in many mathematics competitions. In 1990, he was selected for training at NUS for the International Mathematics Olympiad.

In 1992, he was chosen to represent Singapore in the 33rd IMO held in Moscow, Russia, and he returned with a bronze medal. This was followed in subsequent years with another bronze in Istanbul, Turkey and a silver in Hong Kong. Besides the IMO, he also won a gold at the Asia-Pacific Mathematical Olympiad as well as obtained a perfect score for both the American High School Mathematics Examination (AHSME) and the Australian Mathematics Competition.

Besides his interest in mathematics, Siu Taur was also the chairman of the RI Science Club as well as a member of the RJC canoe team. He also likes to play and watch soccer. Siu Taur is currently waiting to serve national service. In IMO '94 held in Hong Kong, Siu Taur presented a remarkable solution and scored full mark for one of the contest problems. Above is his original solution to the problem in IMO '94.