36th International Mathematical Olympiad

The 36th International mathematical Olympiad (IMO) was held in Toronto, Canada from 13 to 25 July 1995. There were a total of 412 contestants from 73 participating countries. The Singapore National Team to the 36th IMO consisted of the following:

Team Leader:

Dr. Chua Seng Kiat
(National University of Singapore)

Deputy Leader:

Mr. Tai Thiam Hoo
(Raffles Junior College)

Contestants:

Davin Chor Han Ping
(Raffles Junior College)

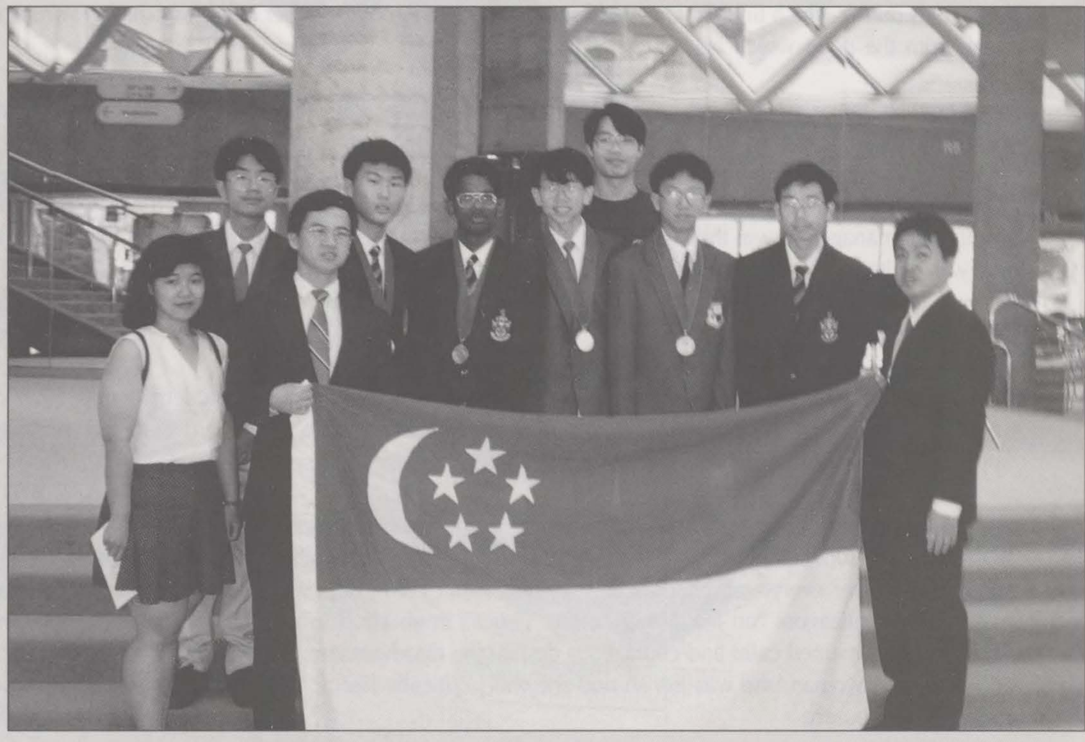
Koh Yi-Huak
(Hwa Chong Junior College)

Jeffrey Pang Chin How
(Anderson Secondary School)

Thevendran Senkodan
(Raffles Institution)

Tay Wee Peng
(Raffles Junior College)

Wee Hoe Teck
(Hwa Chong Junior College)



The National Team with two local hosts

The IMO contest was held on the mornings of 19 and 20 July. The contestants were given four and a half hours to solve 3 problems on each day. Every year prior to the IMO contest, the participating countries are invited to submit problems to the organising committee, which subsequently proposes about 30 preliminary problems. The team leaders from participating countries form the international Jury upon arrival at the contest venue and the 6 problems are decided from these preliminary problems by the International Jury.

At the IMO, only individual awards are given. The participating countries would make an unofficial ranking based on the total number of awards won by each national team which consists of not more than 6 contestants. Besides gold, silver and bronze medals, honourable mentions are awarded to those who do not qualify for the medals but who score full marks for at least one problem. The International Jury follows the general guide lines to decide the award of the medals. The total number of medals awarded each year should not exceed half the number of contestants and the numbers of golds, silvers and bronzes awarded are approximately in the ratio of 1:2:3. In other words, only about one in 12 contestants will receive a gold, one in 6 a silver, and one in 4 a bronze.

This year a total of 30 golds, 71 silvers and 100 bronzes were awarded. Our national team has done well with Davin and Hoe Teck each bagging a silver, Jeffrey and Thevendran each a bronze, and Yi-Huak received honourable mention. Unofficially, Singapore ranked 26 among the 73 participating countries.

IMO 1995 Problems (Day 1)

1. Let A, B, C and D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at the points X and Y. The line XY meets BC at the point Z. Let P be a point on the line XY different from Z. The line CP intersects the circle with diameter AC at the points C and M, and the line BP intersects the circle with diameter BD at the points B and N. Prove that the lines AM, DN and XY are concurrent.

2. Let a, b and c be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)} \quad 3/2.$$

3. Determine all integers $n > 3$ for which there exist n points $A_1, A_2, \ldots, A_n$ in the plane, and real numbers $r_1, r_2, \ldots, r_n$ satisfying the following two conditions:
   (i) no three of the points $A_1, A_2, \ldots, A_n$ lie on a line;
   (ii) for each triple i, j, k ($1 \quad i < j < k \quad n$) the triangle $A_iA_jA_k$ has area equal to $r_i + r_j + r_k$.

IMO 1995 Problems (Day 2)

4. Find the maximum value of x_0 for which there exists a sequence of positive real numbers $x_0, x_1, \ldots, x_{1995}$ satisfying the two conditions:
   (i) $x_0 = x_{1995}$;
   (ii) $x_{i-1} + \dfrac{2}{x_{i-1}} = 2x_i + \dfrac{1}{x_i}$
   for each $i = 1, 2, \ldots, 1995$.

5. Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$ and $DE = EF = FA$, and $\angle BCD = \angle EFA = 60^\circ$. Let G and H be two points in the interior of the hexagon such that $\angle AGB = \angle DHE = 120^\circ$. Prove that

$$AG + GB + GH + DH + HE \quad CF.$$

6. Let p be an odd prime number. Find the number of subsets A of the set $\{1, 2, \ldots, 2p\}$ such that
   (i) A has exactly p elements, and
   (ii) the sum of all the elements in A is divisible by p.