The following is extracted from the website

**http://www.cyberway.com.sg/~kslow/1997.htm**

It was a project undertaken by Secondary Two Gifted Education Programme pupils from Anglo-Chinese School (Independent) under the supervision of their mathematics teacher Mr Low Kok Soon. The page is designed by Alvin Yuan and Ivan Xiao.

## 1, 2, ..., 100*

1997 can be used to form the numbers 1 to 100 by using the digits 1, 9, 9, 7, in that order, the four basic operations (+, -, x, ÷), factorial notation, square root and exponent notations and other non-numerical mathematical symbols.

For example,

$1 = 1 + (9 - 9) \times 7$, $22 = (1 + \sqrt{9})! - 9 + 7$, $44 = -19 + 9 \times 7$,

$96 = (-1)^9 + 97$, $100 = (1 + 9)^{9-7}$. Readers are encouraged to make the complete list.

## Prime numbers and 1997

Firstly 1997 is a prime. From the numbers 1, 9, 9, 7 we can get many other primes, such as: 7, 17, 19, 71, 79, 97, 179, 197, 199, 719, 919, 971, 991, 997, 1979, 7919, 9719, 9791.

But it doesn't just stop there. By using the values of the digits in 1997, other prime numbers can also be obtained besides those mentioned above.

For example:

$1 + 9 + 97 = 107$,

$1 + 99 + 7 = 107$,

$(1 \times 9 \times 9 \times 7) - (1 + 9 + 9 + 7) = 567 - 26 = 541$,

$1 \times 9 \times 9 \times 7 = 567$ (note that the digits are consecutive too!),

$1 \times 9 \times 9 \times 7 + 1 + 9 + 9 + 7 = 593$,

$1997 - (1 \times 9 \times 9 \times 7) - (1 \times 9 \times 9 \times 7) = 863$

Further, from the digits of the primes 199, 19 and their twins 197, 17, two more primes can be formed: 19919 & 19717.

## Sums with patterns

We have also observed the following:

$197 + 1997 = 2194$

$197 + 1997 + 19997 = 22191$

$197 + 1997 + 19997 + 199997 = 222188$

$197 + 1997 + 19997 + 199997 + 1999997 = 2222185.$

Will the subsequent sums constructed in the above manner bear similar patterns (with a chain of 2's at the begining)?*

You are welcome to visit the website for more details and more fun!

* **Editor's note:** The answer is YES to the above question.

Consider taking the sum of $k - 1$ numbers as described above, which is $\sum_{n=2}^{k}(2.10^n - 3)$. One obtains

$$\begin{aligned}\sum_{n=2}^{k}(2.10^n - 3) &= \sum_{n=2}^{k}2.10^n - 3(k-1)\\ &= 2.10^2.\frac{10^{k-1}-1}{10-1} - 3(k-1)\\ &= 2.10^2.\frac{99\ldots9}{9} - 3(k-1)\\ &= 2.10^2.\underbrace{11\ldots1}_{(k-1)\text{ times}} - 3(k-1)\\ &= \underbrace{22\ldots2}_{(k-1)\text{ times}}00 - 3(k-1)\end{aligned}$$

* The Editor has also received a project on such construction from Mr Lim Teck Yong of Anderson Secondary School