The 38th International Mathematical Olympiad

Mar del Plata, Argentina

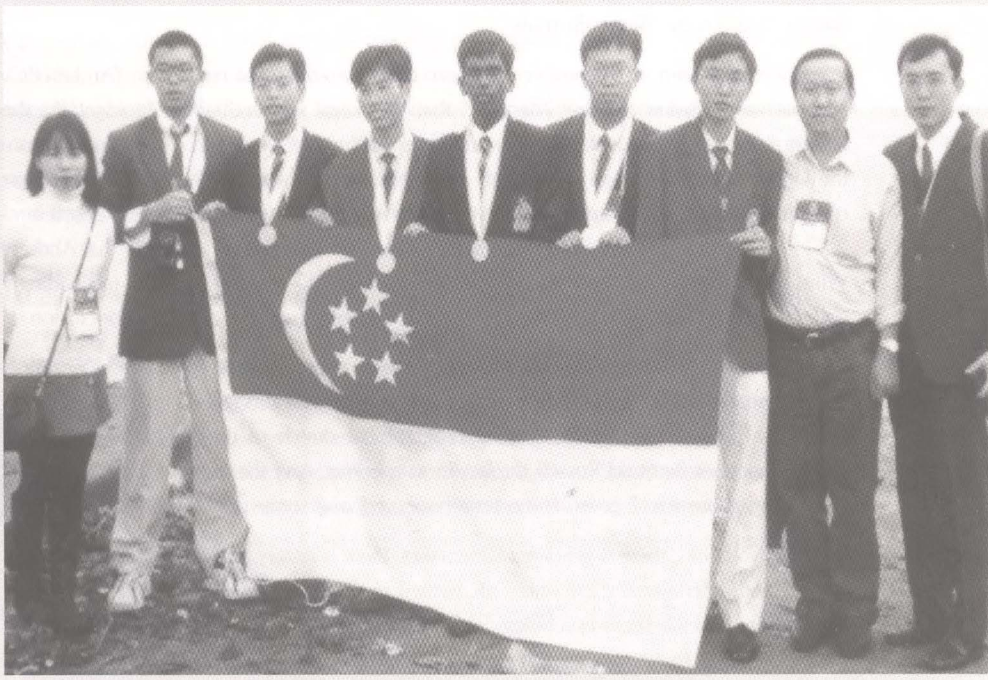
Reported by

Roger Poh and Shaw Swee Tat

The 38th International Mathematical Olympiad was held at Mar del Plata, Argentina from 18 – 31 July, 1997. A total of 460 participants from 82 countries took part in this annual competition for non-tertiary students. Contestants were required to solve six problems in two four and a half hours sessions. Gold, silver and bronze medals were then awarded to deserving students based on their individual scores.

The Singapore team won a total of 4 bronze medals and a honourable mention. While the total score of the team improved from last year's to 88 marks, the unofficial ranking of the team slipped to 40th.

The team was led by team leader Dr Roger Poh Kheng Siong (Mathematics Department, National University of Singapore) and deputy leader, Mr Shaw Swee Tat (National Junior College). The team comprises:



From left to right: Miss Liu Chia Yu (Argentina guide), Yeo Keng Hee, Lin Shaowei, Huah ChengJiann, Thevendran Senkodan, Jeffrey Pang Chin How, Lim Yee Fun, Dr Roger Poh, Mr Shaw Swee Tat.

Thevendran Senkodan	Raffles Junior College	Bronze Medal
Pang Chin How, Jeffrey	Hwa Chong Junior College	Bronze Medal
Huah Cheng Jiann	Victoria Junior College	Bronze Medal
Lin Shaowei	Raffles Institution	Bronze Medal
Lim Yee Fun	Victoria Junior College	Honourable Mention
Yeo Keng Hee	Hwa Chong Junior College	

The 39th International Mathematical Olympiad will be held in Taipei, Republic of China next year.

First day (July 24, 1997)

1. In the plane the points with integer coordinates are the vertices of unit squares. The squares are coloured alternately black and white (as on a chessboard).

   For any pair of positive integers m and n, consider a right-angled triangle whose vertices have integer coordinates and whose legs, of lengths m and n, lie along edges of the squares.

   Let S_1 be the total area of the black part of the triangle and S_2 be the total area of the white part. Let

   $$f(m,n) = |S_1 - S_2|.$$

   (a) Calculate $f(m,n)$ for all positive integers m and n which are either both even or both odd.

   (b) Porve that $f(m,n) \le \frac{1}{2} \max\{m, n\}$ for all m and n.

   (c) Show that there is no constant C such that $f(m,n) < C$ for all m and n.

2. Angle A is the smallest in the triangle ABC.

   The points B and C divide the circumcircle of the triangle into two arcs. Let U be an interior point of the arc between B and C which does not contain A.

   The perpendicular bisectors of AB and AC meet the line AU at V and W, respectively. The lines BV and CW meet at T.

   Show that

   $$AU = TB + TC.$$

3. Let $x_1, x_2, \ldots, x_n$ be real numbers satisfying the conditions:

   $$|x_1 + x_2 + \ldots + x_n| = 1$$

   and

   $$|x_i| \le \frac{n+1}{2} \quad \text{for} \quad i = 1, 2, \ldots, n.$$

   Show that there exists a permutation $y_1, y_2, \ldots, y_n$ of $x_1, x_2, \ldots, x_n$ such that

   $$|y_1 + 2y_2 + \ldots + ny_n| \le \frac{n+1}{2}.$$

Each problem is worth 7 points.

Time: $4\frac{1}{2}$ hours.

Second day (July 25, 1997)

4. An $n \times n$ matrix (square array) whose entries come from the set $S = \{1, 2, \ldots, 2n - 1\}$ is called a *silver* matrix if, for each $i = 1, \ldots, n$, the ith row and the ith column together contain all elements of S. Show that

   (a) there is no silver matrix for $n = 1997$;

   (b) silver matrices exist for infinitely many values of n.

5. Find all pairs (a, b) of integers $a \ge 1$, $b \ge 1$ that satisfy the equation

   $$a^{b^2} = b^a.$$

6. For each positive integer n, let $f(n)$ denote the number of ways of representing n as a sum of powers of 2 with nonnegative integer exponents.

   Representations which differ only in the ordering of their summands are considered to be the same. For instance, $f(4) = 4$ because the number 4 can be represented in the following four ways:

   $$4;\ 2 + 2;\ 2 + 1 + 1;\ 1 + 1 + 1 + 1.$$

   Prove that, for any integer $n \ge 3$,

   $$2^{n^2/4} < f(2^n) < 2^{n^2/2}.$$

Each problem is worth 7 points.

Time: $4\frac{1}{2}$ hours.