BOOK • REVIEW

Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being

by Lakoff and Nunez

The only National Library copy of this book is available in Tampines Regional Library.

The two authors – Lakoff, a linguist and Nunez, a psychologist – purport to introduce a new field of study, i.e. "mathematical idea analysis", with this book. By "mathematical idea analysis", they mean to give a scientifically plausible account of mathematical concepts using the apparatus of cognitive science. This approach is meant to be a contribution to academics and possibly education as it helps to illuminate how we cognitise mathematical concepts, which are supposedly undecipherable and abstruse to laymen. The analysis of mathematical ideas, the authors claim, cannot be done within mathematics, for even metamathematics – recursive theory, model theory, set theory, higher-order logic – still requires mathematical idea analysis in itself! Formalism, by its very nature, voids symbols of their meanings and thus cognition is required to imbue meaning. Thus, there is a need for this new field, in which the authors, if successful, would become pioneers.

My questioning tone of the above paragraph belies a certain suspicion as to the credibility of this work. The assumption that mathematics cannot describe itself can be attacked: language, a symbol system, albeit a complex one, is used to describe itself so why can't mathematics – a symbol system with way less constrains – do so too? Without the premise I just used above, much of linguistics would not exist. Basically this means that we cannot continue the programme of formalism, which, anyway, Godel has already shown to be untenable.

More significantly, the book's weakness is in its lack of scientific evidence. The authors explain mathematical concepts using cognitive scientific concepts like conceptual metaphors ("Numbers are object collections"), conceptual blends (The number line is a blend of numbers and the metaphor "numbers are points on a line".) and container schemas (basically giving us the intuitive properties of sets). The existence of these concepts are supported by evidence in linguistics, neurobiology and other cognitive sciences. However, all these explanations and hypotheses are simply stated without evidence that they have been tested scientifically. They appear to be simply the authors' speculations. This casts doubt on the credibility of the book's work.

The book however does a good job in explaining all the mathematical ideas, albeit in a technical way, in the manner in which most mathematicians conceptualize them. It thus contributes to education as many mathematics student and laymen do not have these specialized, idealized conceptualizations but have intuitive ones in their cognitive vocabulary. The exact nature of these conceptualizations are thus clarified, though this cannot be said to be of significant academic contribution as they are already known, though not published, within the mathematical community.

The best idea in the book is perhaps the "Basic Metaphor of Infinity" or BMI for short. The BMI basically says that infinite processes are conceptualized in terms of finite processes. Since finite processes have final resultant states, the entailment which results from the mapping of a concept in the source domain – here "finite processes" – to the target domain – here "infinite processes" – results in the concept of a resultant of potential infinities, that is, an actual infinity. This explanation is, on the surface, satisfactory but like many of the other metaphors, e.g. "Propositions are classes", rely on language – and thus our intuitive understanding of concepts written as words – and so do not reach the level of precision the authors hope, I believe, to attain. Thus this explanation does the solve the mystery of actual infinity which has befuddled mathematicians and philosophers since antiquity. What is significant here, however, is the clever use of the BMI in "proving" the existence of many concepts and in giving their definitions. Some of these concepts include the set of natural numbers, the set of real numbers, hyperreals, infinitesimals, points and lines, and the point at infinity both in projective geometry and inversive geometry. It is amazing how the authors manage to tie in so many seemingly diverse concepts as special cases of a single conceptual metaphor.

A massive case study on Euler's equation lies at the back of the book. To explain the cognition behind this equation, a cognitive analysis is given of much of elementary mathematics, the area of mathematics usually termed as "precalculus". A close study of this section of the book would be useful for mathematics students and teachers.

In the end, this book should be considered a work in the philosophy of mathematics rather than in cognitive science. The fact that this book has been used as one the texts in the first-year philosophy module in the National University of Singapore for a few years in the recent past supports my above view. The perspective this book offers is that mathematics is not universal or delivers *a priori* truths, as is the view of the Platonists, but rather is embodied in man's conceptual systems and is shaped by the views of the philosophers of mathematics. The qualities of the mathematics that is mainstream in the modern world, like if-and-only-if definitions and the need for foundations, is a result of the views of the influential Greek philosophers, including Plato and Aristotle, whose thought is still studied today. (Their influence on philosophy can be compared to that of Shakespeare on literature.)

Which brings us to the question: how would mathematics have been if the views of philosophers of other cultures have shaped mathematics instead?

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