BOOK • REVIEW

Kepler's Conjecture: How Some of the Greatest Minds in History Helped Solve One of the Oldest Math Problems in the World

by George G. Szpiro, Wiley, Hoboken, New Jersey

The subtitle of this book is a mouthful: *"How some of the greatest minds in history helped solve one of the oldest math problems in the world."* The Kepler associated with this conjecture is the famous Kepler of "Kepler's Laws" in the mechanics of the solar system. What is less well-known (at least to the non-mathematician) is that Kepler wrote in 1611 a booklet that deals with more down to earth phenomena like the shapes of snowflakes, honeycombs and seeds of pomegranates. For all its geometric observations, this booklet would have faded into the mist of history had it not contained a claim (without proof) that the tightest (densest) way to pack equal spheres in (infinite) space is to surround each sphere with twelve other spheres in a certain way – a "fact" that the fruit seller or goods packer will certainly attest to.

The part about the twelve spheres touching a central sphere is sometimes called the "kissing" problem, and the part about "densest" packing is usually referred to as "Kepler's Conjecture". Here "density" is the "global" density of the packing, namely the limit of the ratio of the total volume of all the spheres to the volume of the region containing these spheres as the region expands to fill up all of space.

The (global) density of Kepler's dense packing of spheres in three dimensions is 74.05%. If someone came up with a packing with density less than 74.05%, then Kepler's Conjecture would have been disproved. Thus one approach to the Conjecture is to search for upper bounds for any arbitrary packing. Around 1929, the Stanford mathematician Blichfeldt found the first upper bound for any arbitrary packing, his best bound being 83.5%, which is still a long way away from the Kepler's density. Not until 1947 was this bound reduced to 75.46 by the Scottish mathematician Rankin. From then on, it became a race to beat the existing record. And like any race, it is a race for fame and recognition.

This book traces in a captivating and amusing way the false starts, failures and efforts of various mathematicians trying to prove Kepler's claims, starting from the two-dimensional version and culminating in the full-blown general version in three dimensions, and along the way, offering insights into the generalized version in higher dimensions in scattered cases. There is a special, somewhat more "orderly" version ("lattice packing") of Kepler's Conjecture that was solved by none other than the "Prince of mathematicians" (the German mathematician Gauss) himself, using purely number theoretic methods that built on fundamental concepts introduced by the French mathematician Lagrange (hailing originally from Italy). A long list of both familiar as well as lesser known names are posted along the path to the solution of the conjecture in two dimensions: among others, Newton, Gregory, Dürer, Lagrange, Seeber, Gauss, Thue, Minkowski, Kershner, Segré, Mahler, Fejes-Tóth.

In a landmark address at the Paris Congress of the International Mathematical Union in 1900, one of the greatest mathematician of the twentieth century, David Hilbert, included a more general problem related to Kepler's Conjecture as one part of Problem Number 18 in his famous list of 23 then unsolved problems. So important and hitherto intractable were these problems (and many still are) that the solution of any one of the Hilbert Problems will not only ensure academic tenure but also a place in the pantheon of mathematics.

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As more and more books (of which this book is one of the most recent) are written to popularize advanced mathematics, the lives of mathematicians are coming under the scrutiny of the general public. In this book, you will also learn about the more earthly struggles, fortunes and misfortunes of the protagonists drawn into the drama surrounding Kepler's Conjecture. Mathematicians are not just cold thinking machines; they are also subject to the foibles of life and nature. You may be heartened to read the fairy tale like story of Stanford mathematician Blichfeldt from farmhand to draftsman to surveyor to head of the department of mathematics. One is tempted to romanticize that the compulsive search for mathematical truth must be a reflection of an inner attraction to moral truth. Unfortunately, this view has a (what mathematicians would call) counterexample (more than one, in fact) in a brilliant mathematician who believed in Nazi morality deep enough to support the irrational race policy of Hitler in the universities.

The author (George Szpiro) is no ordinary paparazzi – he is a mathematician turned scientific journalist. There is enough detailed mathematics in the main text and in 47 pages of appendices to keep the mathematically minded reader mentally active and challenged. It is not easy to visualize the geometric concepts of Voronoi cells and Delaunay triangulations that are used to inch forward. It would have been a triumphant *tour de force* even if it took sheer mental brute force to tackle the finite, though large, number of cases that remained to be eliminated. The scheme which Thomas Hales hatched to confirm Kepler's guess had to be implemented with the aid of a computer (and the programming skills of his student Samuel Ferguson). The crux of the mathematics that Hales used is not more sophisticated than the methods of linear programming. The total number of pages he needed is large (250 pages), but not really forbiddingly large, but what is perhaps not so graceful or aesthetic to the mathematician is the 3 gigabytes of data that go into the programs and output that are needed to deliver the *coup de grace.* Hales and presumably some others have checked the programs, but it may require more time before his computer-aided proof reaches the same status as that of the four-color problem (by now, the four-color theorem).

This book contains many details, both of a mathematical and a non-mathematical nature. The general reader will probably lose track of the names of the players in this drama and of the chain of ideas and methods leading to the final ones. On reading the book again, you will be surprised to find out how good or how bad your retentive memory is. If you have the stomach for mathematical details or if you are prepared to skim over them, the book reads like a thrilling novel with lots of tantalizing subplots. With enough patience (and endurance), you will find your mental horizons, as they say, greatly widened.

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