If $m = 2$ and exactly two of a, b, c, d, e is even, then $a^2 + \cdots + e^2 \equiv 3 \equiv f^2 \pmod 8$, a contradiction.

So in all cases, we have $8 \mid abcdef$. So $d = 24$.

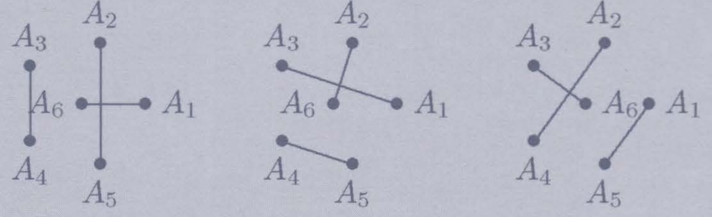
10. (Ukrainian Mathematical Olympiad, 2004) Let $A_1, A_2, \ldots, A_{2004}$ be the vertices of a convex 2004-gon(i.e., a polygon with 2004 sides). Is it possible to mark each side and each diagonal of the polygon with one of 2003 colours in such a way that the following two conditions hold:

(1) there are 1002 segments of each colour;

(2) if an arbitrary vertex and two arbitrary colours are given, one can start from this vertex and, using segments of these two colours exclusively, visit every other vertex only once?

Solution by the editor. The actual location of the vertices are not important. The problem is about colouring the edges of the complete graph on n vertices, n even. We shall place vertices $A_1, \ldots, A_{n-1}$ evenly on the circumference of a circle and A_n at the centre.

Colour the edge A_nA_i and all the edges perpendicular to it with colour i. This certainly satisfies condition (1). To satisfy condition (2), we need to show that for any two colours i, j, the edges with these two colours form a cycle. (The figure below shows for $n = 6$, the edges with colours 1, 2, 3. It's easy to see that any two of the three sets of edges form a cycle.



Without loss of generality, we only need to show this for the case of colours 1 and i. In the subgraph formed by these two sets of edges, every vertex is of degree 2. Thus it is the union of edge disjoint cycles. To show that it is a single cycle, we only need to show that the subgraph is connected. For any vertex A_a, the edge A_aA_{2j-a}, (here the subscripts are taken mod $n-1$), is perpendicular to A_nA_j. Thus it has colour j. Thus the edge A_aA_{2-a} is of colour 1 and the edge $A_{2-a}A_{2i-2+a}$ is of colour i. Thus A_a is connected to $A_{2(i-1)+a}$. Hence A_1 is connected to $A_{2p(i-1)+1}$, $p = 1, \ldots, n-2$. But these vertices are distinct since $2j+1 \equiv 2k+1 \pmod{n-1}$ implies that $2(j-k) \equiv 0 \pmod{n-1}$. But $n-1$ is odd. Therefore $j = k$ if they are both $\leq n-1$. Hence the subgraph is connected are required.

Mathematical Talent — Looking at "mediocrity" with an ordinary mind

by
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Proof is not a play on mathematics, but one where the backdrop is steeped in the activities of mathematicians. For this reason, an audience with no mathematical knowledge can still enjoy the drama filled with family affection, friendship and love — the emotional conflict, the self-reproach, the mutual trust and the excitement that go on in the inner world of the characters, reflected as joy, anger, grief and happiness that rise and fall as the play unfolds. Among these, the intimate relationship between the father and the daughter is the most crucial: the father's expectation of and his love for the daughter, the daughter's affectionate concern about the father as well as self-blame on her sporadic resentfulness caused by the pressure accumulated throughout the long years in taking care of him. This is indeed touching.

Since the story develops with the father/daughter relationship as the central theme, the proof of that important theorem in the play becomes all the more significant. The title of the play *Proof* indicates not only the proof of the theorem, but also whether it is possible (or necessary?) to prove who the genuine author of the extremely creative proof of the theorem is. Outside the mathematical domain, there are things which need no rigorous proof and yet are acceptable by most people. Excessive demand of a proof may be a disservice! In fact, even in mathematics, does the proof of a mathematical result serve solely as a verification of the validity of the result?

Many people when watching the play notice the almost indistinguishable difference between a mathematical genius and a psychotic patient. However, I would like to take a different point of view of the opinion expressed by some characters in the play (opinion perhaps shared by many mathematicians as well): we all bear the pressure of accepting "mediocrity" with resignation. This is particularly so in the case of mathematics, where it is generally thought that a talented mathematician should shine at an early age. As a mathematician gets older, he would feel that he is past his prime and regret that he is approaching the twilight years of his career. As a matter of fact, in this respect mathematics fares better than other forms of art. (Mathematics, is it not also a form of art?) Any bit of work in mathematics, no matter how unimportant or how unknown it may be to later generations, can in some ways push forth the development of the subject. The fruitful results of individuals are assimilated into one single whole; the contribution of an individual melds with this single whole. Throughout the ages there were thousands and thousands of mathematicians, but only a handful of them went down in the annals of history. Among contemporary mathematicians only a minority are recognized as outstanding. Judging from this, should we not look at "mediocrity" with an ordinary mind?

(This article, translated by Fung Kit CHAN, originally appeared in Chinese in the house programme of the play staged by the Hong Kong Repertory Theatre in July 2005 in Hong Kong. The play script of "***Proof***" written by David Auburn won the Pulitzer Prize and the Tony Award in 2001.)