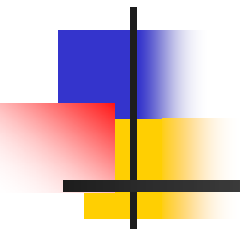
Assessment and Further Generation of Ideas



Overview

- Reinforce the criteria for a good mathematics project brought up in the first session. More details and examples will be given based on previous project competitions.
- Use the criteria as a base from which to generate more ideas for projects that would be deemed ‘mathematical’. Examples from school syllabus questions, from daily life and from undergraduate engineering calculus projects will be given.
- Brainstorm possible projects and discuss these with your partners.

Criteria for a good mathematics project

- Interesting area explored
- Problems that arise are investigated using mathematical reasoning and techniques
- Intelligent and/or innovative use of mathematics to solve the problems
- Some research into existing solutions?
- Written report must be mathematically presented

Criteria for a good mathematics project report

- Area to be covered is clearly stated
- Problems to be solved are clearly stated and proofs of crucial problems are to be complete and mathematical
- Written report must be mathematically presented
- Mathematical thinking must be evident – in choice of problems, solutions, extensions and further questions posed

Interdisciplinary Project Work vs Mathematics Project

- Maths is a servant
- General research techniques
- Read and write interestingly and ‘formally’
- The more, the merrier

- Maths is the king
- Need to be able to think mathematically
- Read and write mathematics
- Better to have a complete solution to a small problem than sketches of many disparate areas
- The posing of questions and extensions (although unsolved) is also very important

Mathematical Proof

- Logical thinking
  - $A \Rightarrow B$ Statement
  - $A \Leftarrow B$ Converse
  - $A' \Leftarrow B'$ Contrapositive
  - $A' \Rightarrow B'$ Inverse
- Conditions for the theorem to be true

Mathematical Proof: Non-example

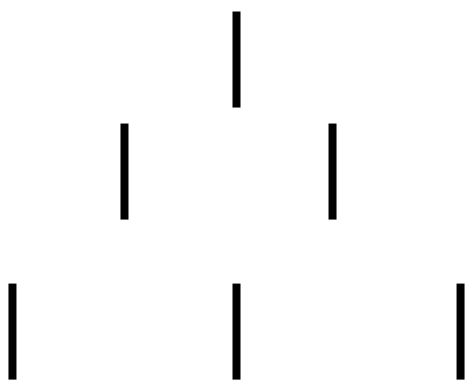
In any game played by two perfectly logical persons making moves alternately, the second person to move will always lose.

Proof (?)

Suppose the second person wins. Then the second person will have a winning strategy. If that is the case, then the first person can use the same winning strategy. Since he begins first, he will implement it and win; contradiction.

Mathematical Proof: Non-example

- **Question:** Are the conditions the same?
- **Counterexample – Sticks Game**:



Sticks game – some research needed

- Project submitted showed good work on exploration and dividing into cases – partial solution.
- Unfortunately, students did not do any research at all – complete solution exists:
  - http://sps.nus.edu.sg/~limchuwe/cgt/cgt2.htm
  - http://www.mps.k12.nf.ca/mathematics/Grassroots/Game%20of%20nim/nim3.htm

Mathematical Thinking

- Problem solving
- Conjecturing
- Creativity and rigour
- Extensions and posing problems

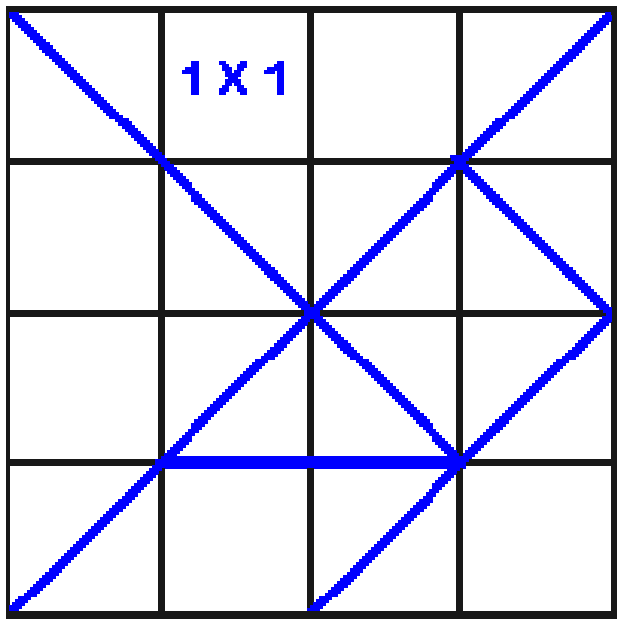
Example 1 -Tangram

- http://www.tangrams.ca/
- Backup
- Solve the problem
- Look for techniques to solve
- Ask more mathematical questions
  - What kind of geometrical figures are possible?
  - What kind of geometrical figures are impossible?

Tangram - What is possible?

- Given an unlimited number of tangram sets each of area 1, can the following geometrical shapes be constructed? **Prove your answer**.
  - A square of area 2.
  - A square of area 3.
- Generalise your observations.

Tangram – Lengths of sides



- 4
- $2\sqrt{2}$
- $\sqrt{2}$
- 2

Example 2 - Sudoku

- Solve the problem
- Look for techniques to solve
  - http://www.math.nus.edu.sg/aslaksen/sudoku
  - Powerpoint slides – acknowledgement to A/P Helmer Aslaksen
- Ask more mathematical questions
  - How to construct
  - How many possibilities
  - Maximum number allowed to erase to ensure unique solution

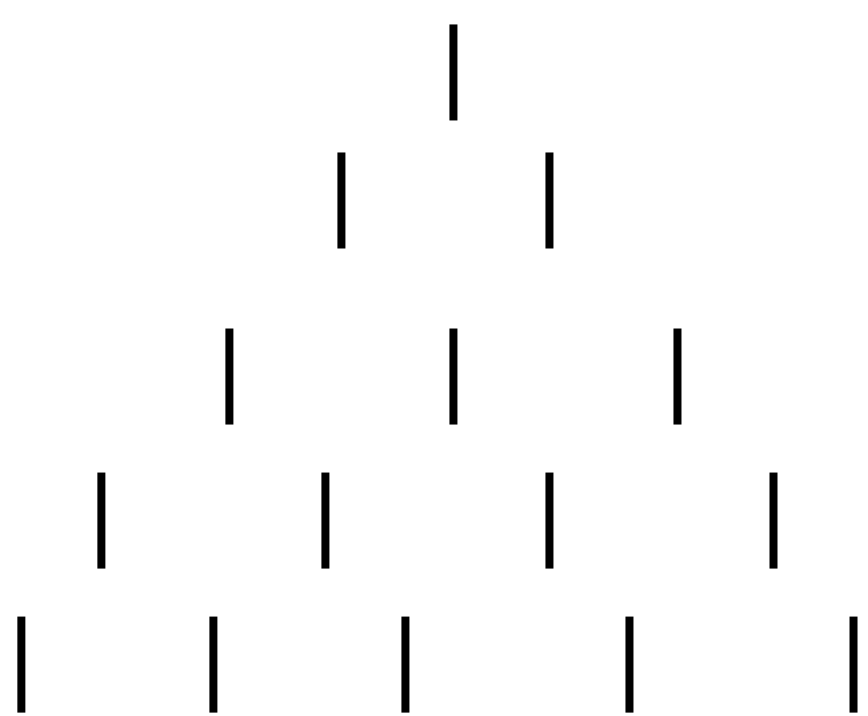
Example 3 – Laws of Indices

- Ask questions about the maths you learn in school
- Learn about the mathematical method
  - Besides content and rigour, a mathematics course also teaches (indirectly, by osmosis) the mathematical method: how to formalize concepts with definitions, work with small examples to gain some intuition, formulate conjectures to capture the intuition, build up to a theorem with useful lemmas, derive special cases as corollaries, explore the limits of the theorem with generalizations and counterexamples and thus construct a theory. (Tay Yong Chiang, 2004)
- Ask how far it can go

Example 4 – Crossing out sticks

- An early project
- Solved partially using case analysis
- A little more research will show that an ingenious complete solution using binary representation exists
  - http://sps.nus.edu.sg/~limchuwe/cgt/cgt2.htm
  - http://www.mps.k12.nf.ca/mathematics/Grassroots/Game%20of%20nim/nim3.htm

Crossing out sticks – last to strike wins



- 001
- 010
- 011
- 100
- 101

001

Example 4 – Crossing out sticks

- However, can document the process of partial solution solved *independently* and later of complete solution.
- Also can further devise variations of the game using the same idea of binary (or ternary?) representation.
- Can devise variations of the game requiring different number of strokes to eliminate the stick.

Example 5 – The most important limit in Calculus

- *Student Research Projects in Calculus*,

  Marcus S. Cohen, Mathematical Association of America (1991)
- Structured project with strong support
- Need not be for competition
- May also be the initial phase of a project to be submitted for competition

Everyday problems

- 4-D Near Miss
  - Why do people continue to play the game when the payout is -$0.37 for a $1 bet?
- Chinese New Year date and calendar
  - http://www.math.nus.edu.sg/aslaksen/calendar/chinese.html
- Car loans
  - Why are car salespersons so eager to offer gifts to encourage customers to take loans?
- Insurance premiums
  - How are these calculated? Should you buy?

Brainstorm possible projects

- Discuss with a partner
- Write down possible project areas on a piece of paper
- Elaborate on one project
- Hand up your pieces of paper