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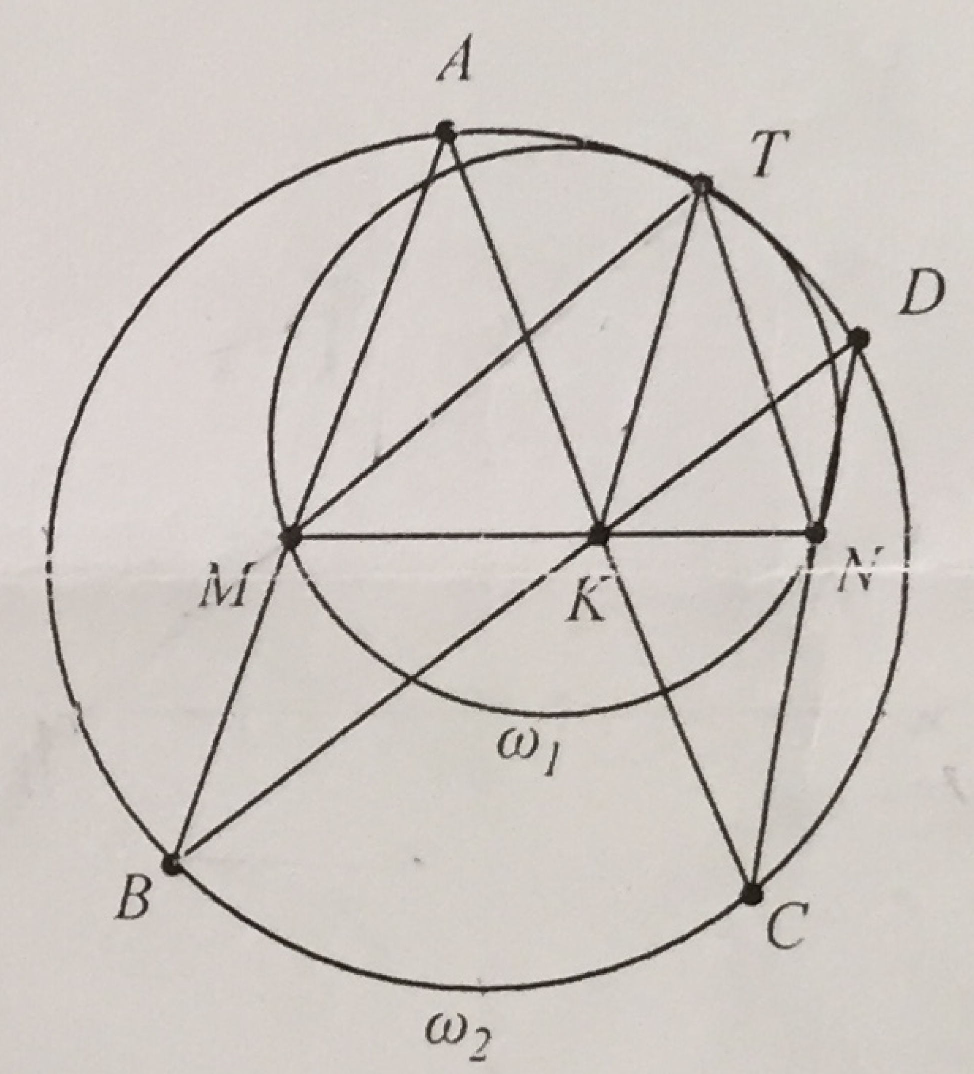
First Day 16th August, 8:00 am ~12:00 noon

Each problem is 15 marks

1. Let n be a given positive integer, and $x_1, x_2, \cdots, x_n$ are real numbers such that the sum $\sum_{k=1}^{n} x_k$ is an integer. Let $d_k = \min_{m\in\mathbb{Z}} |x_k - m|, 1 \le k \le n$. Determine, with proof, the maximum value of the sum $\sum_{k=1}^{n} d_k$.

2. As shown in the figure, circles ω_1 and ω_2 are tangent to each other at the point T. M, N are two distinct points on ω_1 and different from T. AB and CD are two chords of ω_2 passing M, N respectively. Prove that if the segments AC, BD, MN meet at the same point K, then the line TK bisects $\angle MTN$.



3. Let $n \ge 2$,be an integer, and $x_1, x_2, \cdots, x_n$ are positive real numbers such that $\sum_{i=1}^{n} x_i = 1$. Prove that

$$\left(\sum_{i=1}^{n} \frac{1}{1-x_i}\right)\left(\sum_{1\le i<j\le n} x_i x_j\right) \le \frac{n}{2}.$$

4. For 100 straight lines on a plane, let T be the set of all right-angled triangles bounded by some 3 lines. Determine, with proof, the maximum value of $|T|$.