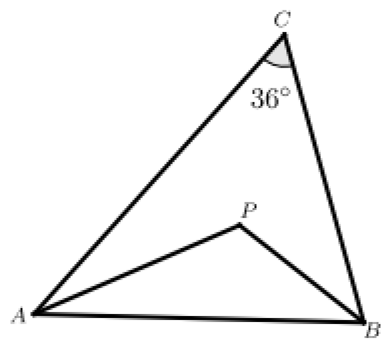
Here are some examples of first round questions for the three sections. These are questions that are based on school mathematics syllabus at the respective levels. A significant number of questions at similar level and format will be featured in the SMO first round competitions.

Junior Section

1. In $\triangle ABC$, $\angle ACB = 36^\circ$ and the interior angle bisectors of $\angle CAB$ and $\angle ABC$ intersect at P. Find $\angle APB$.



(A) 72° (B) 108° (C) 126° (D) 136° (E) None of the above

Comment: This problem involve basic properties of a triangle and the concept of angle bisectors.

2. A painting job can be completed by Team A alone in 2.5 hours or by Team B alone in 75 minutes. On one occasion, after Team A had completed a fraction $\frac{m}{n}$ of the job, Team B took over immediately. The whole painting job was completed in 1.5 hours. If m and n are positive integers with no common factors, find the value of $m + n$.

Comment: This problem involve the solving linear equations in one unknown (including fractional coefficients) prescribed in the Secondary 1 syllabus. The context of the word problem should not be too unfamiliar to students.

Senior Section

1. Let α and β be the roots of the equation $x^2 + x - 3 = 0$. Which of the following is the value of $\alpha^3 - 4\beta^2 + 20$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

Comment: The knowledge about the roots of a quadratic equation is covered in O-level syllabus. In particular, if α and β are the roots of the quadratic equation $f(x) = x^2 + bx + c = 0$, then $f(\alpha) = f(\beta) = 0$, $\alpha + \beta = -b$ and $\alpha\beta = c$. For the given quadratic equation, we have

$$\alpha^2 + \alpha - 3 = 0, \quad (1)$$

$$\beta^2 + \beta - 3 = 0, \quad (2)$$

$$\alpha + \beta = -1, \quad (3)$$

$$\alpha\beta = -3. \quad (4)$$

The challenging part is to manipulate these equations to get the value of $\alpha^3 - 4\beta^2 + 20$. Most good O-level students should be able to do this, though in this example one does not even need to use equation (4).

2. Find the value of $(25 + 10\sqrt{5})^{1/3} + (25 - 10\sqrt{5})^{1/3}$.

Comment: Let $x = (25 + 10\sqrt{5})^{1/3} + (25 - 10\sqrt{5})^{1/3}$. It is assumed in the competition that all answers are integers, so x must be an integer. In fact, any positive power of x will still be an integer. It seems difficult to determine the value of $(25 \pm 10\sqrt{5})^{1/3}$ directly. The key is to see that it might be easier to compute the powers of x.

In this example, the expression suggests that we can try to compute x^3 . Most O-level students should be able to expand $\left((25 + 10\sqrt{5})^{1/3} + (25 - 10\sqrt{5})^{1/3}\right)^3$ using binomial expansion, and simplify the resulting expressions which contain surds. It turns out that $x^3 = 50 + 15x$, or $(x - 5)(x^2 + 5x + 10) = 0$. This equation admits only one real root $x = 5$.

Open Section

1. Find the sum $1\times 2+2\times 3+3\times 4+...+100\times 101$.

Comment:

Most good A-level students should be able to solve Q1. There are two possible methods at the A-levels that can be easily used to solve this problem:-

Method 1: Using the formulae $\sum r$ and $\sum r^2$. Candidates are required to be able to formulate the question into the form that matches the two formulae.

Method 2: Using method of difference, candidates must be able to see that each term can be expressed as a difference of a cubic polynomial.

Although both methods are workable, in view of the competition environment that no calculators are allowed, Method 2 is a much quicker one. However, most good A-level students should be able to use Method 1, although it will take a slightly longer time in computation.

2. Let $a_1, a_2, a_3, ...$ be an arithmetic progression with $a_1 > 0$ and $3a_8 = 5a_{13}$. Let $S_n = a_1 + a_2 + ... + a_n$ for all positive integer n. Find the integer n such that S_n has the largest possible value.

Comment:

The content knowledge is definitely covered in A-level syllabus. Formulae for the sum of the first n terms and the nth term of an arithmetic progression should be easy for an average A-level student. The slightly challenging part is the formulating of the equation $3a_8 = 5a_{13}$, and what information does "S_n having the largest possible value" tell us? One would consider this as a higher order thinking question at A-levels. Even slightly above average students without Olympiad training are able to solve this question.