The International Mathematical Olympiad

Background

The International Mathematical Olympiad (IMO) is an annual competition for students under 20 years of age and who have not studied at a tertiary institution. The IMO began in 1959 as a regional competition among East European countries. Since then, the number of participating countries has grown to about forty.

The competition consists of two $4\frac{1}{2}$-hour papers, each with three problems. Each problem carries 7 points, so the maximum is 42 points. Gold, silver and bronze medals are awarded to about half of the participants; usually, about one-twelveth of the participants get gold medals. Special prizes are sometimes awarded for particularly elegant or mathematically interesting solutions.

Each country can send in a team of at most six members. The IMO is an individual event, so there are no team prizes. However, team rankings are usually compiled unofficially by adding up the scores of team members.

Table 1 ranks the participating countries according to their total scores for five previous Olympiads. Table 2 lists the results of last year's competition.

IMO 1986: An example

These are the six problems for IMO 1986:

1. Let d be any positive integer not equal to 2, 5 or 13. Show that one can find distinct a, b in the set $\{2, 5, 13, d\}$ such that $ab - 1$ is not a perfect square.

2. A triangle $A_1A_2A_3$ and a point P_0 are given in the plane. We define $A_s = A_{s-3}$ for all $s \geq 4$. We construct a sequence of points P_1, P_2, P_3, ... such that P_{k+1} is the image of P_k under rotation with center A_{k+1} through angle 120° clockwise (for $k = 0, 1, 2, \ldots$). Prove that if $P_{1986} = P_0$ then the triangle $A_1A_2A_3$ is equilateral.

Table 1. Rankings of total scores in IMO

		1959	1982	1983	1984	1985
1.	Algeria	—	—	30	—	—
2.	Australia	—	20	19	15	—
3.	Austria	—	16	22	16	—
4.	Belgium	—	24	26	23	15
5.	Brazil	—	20	20	18	10
6.	Bulgaria	4	9	9	2	4
7.	Canada	—	17	14	20	9
8.	Columbia	—	26	28	21	—
9.	Cuba	—	25	24	22	—
10.	Cyprus	—	—	—	26	—
11.	Czechoslovakia	3	7	8	13	—
12.	F.R.G.	—	1	1	9	—
13.	G.D.R.	6	3	12	8	—
14.	Finland	—	8	13	28	17
15.	France	—	15	10	12	6
16.	Great Britain	—	10	11	6	7
17.	Greece	—	23	17	19	13
18.	Hungary	2	6	3	4	3
19.	Israel	—	15	16	—	11
20.	Italy	—	—	32	31	—
21.	Kuwait	—	29	31	30	—
22.	Luxembourg	—	13	29	—	—
23.	Mongolia	—	22	—	10	—
24.	Morocco	—	—	25	23	—
25.	Netherland	—	14	7	17	—
26.	Poland	5	13	15	11	—
27.	Romania	1	11	5	3	1
28.	Sweden	—	19	21	25	14
29.	U.S.S.R.	—	2	4	1	5
30.	Spain	—	—	23	27	—
31.	Tunisia	—	28	27	29	—
32.	Turkey	—	—	—	—	16
33.	U.S.A.	—	3	2	4	2
34.	Venezuela	—	27	—	—	—
35.	Vietnam	—	5	6	7	—
36.	Yugoslavia	—	12	18	14	—

Table 2. The 27th IMO, Warsaw, Poland (July 9-10, 1986)

	COUNTRY	SCORE	NUMBER OF MEDALS Gold	Silver	Bronze
1.	Algeria	80	—	—	2
2.	Australia	117	—	—	5
3.	Austria	127	—	2	2
4.	Belgium	75	—	1	1
5.	Brazil	69	1	—	—
6.	Bulgaria	161	1	3	2
7.	Canada	112	—	2	1
8.	China	177	3	1	1
9.	Columbia	58	—	—	—
10.	Cuba	51	—	—	—
11.	Cyprus	53	—	1	—
12.	Czechoslovakia	145	—	3	3
13.	F.R.G.	196	2	4	—
14.	G.D.R.	172	1	3	2
15.	Finland	60	—	—	1
16.	France	131	1	1	2
17.	Great Britain	141	—	2	3
18.	Greece	63	—	—	2
19.	Hungary	151	1	2	2
20.	Iceland	37	—	—	—
21.	Israel	119	—	2	2
22.	Italy	49	—	—	1
23.	Kuwait	48	—	—	—
24.	Luxembourg	22	—	—	—
25.	Mongolia	54	—	—	—
26.	Morocco	90	—	1	1
27.	Norway	68	—	1	—
28.	Poland	93	—	—	2
29.	Romania	171	2	2	1
30.	Sweden	57	—	—	1
31.	U.S.S.R.	203	2	4	—
32.	Spain	78	—	1	1
33.	Tunisia	85	—	—	1
34.	Turkey	55	—	—	—
35.	U.S.A.	203	3	3	—
36.	Vietnam	146	1	2	2
37.	Yugoslavia	84	—	—	2

3. To each vertex of a regular pentagon an integer is assigned in such a way that the sum of all the five numbers is positive. If three consecutive vertices are assigned the numbers x, y, z respectively and $y < 0$ then the following operation is allowed: the numbers x, y, z are replaced by $x + y$, $-y$, $z + y$ respectively. Such an operation is performed repeatedly as long as at least one of five numbers is negative. Determine whether this procedure *necessarily* comes to an end after a finite number of steps.

4. Let A, B be adjacent vertices of a regular n-gon ($n \geq 5$) in the plane having center at O. A triangle XYZ, which is congruent to and initially coincides with OAB, moves in the plane in such a way that Y and Z each trace out the whole boundary of the polygon, X remaining inside the polygon. Find the locus of X.

5. Find all functions f, defined on the non-negative real numbers and taking non-negative real values, such that:

   (i) $f[xf(y)] \cdot f(y) = f(x + y)$ for all $x, y \geq 0$,

   (ii) $f(2) = 0$,

   (iii) $f(x) \neq 0$ for $0 \leq x < 2$.

6. One is given a finite set of points in the plane, each point having integer coordinates. Is it always possible to color some of the points in the set red and the remaining points white in such a way that for any straight line L parallel to either one of the coordinate axes the difference (in absolute values) between the numbers of white points and red points on L is not greater than 1? Justify your answer.

IMO 1988: Australia

Singapore will send a team to IMO 1988, which will be held in Australia. (Cuba will host IMO 1987 in July.) The following is a brief description of the 13-day program.

In addition to the student participants, a team has a leader and (usually) a deputy leader. On Day 1, the leader will arrive in Canberra, while the deputy and students arrive in Sydney. The students will tour Sydney, and leave for the competition venue (Goulburn) on Day 4. The opening ceremony will be on Day 5 in Canberra (a short ride from Goulburn).

The two examinations will be held on the mornings of Days 6 and 7. After the second examination, the students will go to Canberra, and stay there for the rest of the trip. They will have excursions, barbecues and other social activities before the awards ceremony on Day 12. Meanwhile (from Day 1 to Day 12), the leader will be in Canberra, setting questions and marking scripts.

On Day 13, the teams will be driven back to Sydney for their departure.

Preparation of Singapore's team

Based on a selection test held by the Ministry of Education and the Singapore Mathematical Society in December 1986, 48 students have been selected for training for IMO 1988. These students are now secondary four and first-year junior college students.

So far, the training consists of weekly assignments, lectures, and tutorials. In June, there will be a training camp; as part of the camp's programme, the trainees will participate in the Society's annual Interschool Mathematical Competition. Based on the results of the competition, the number of trainees will be reduced to about 20. These 20 may include students who are not among the 48 previously selected, but who do well in the interschool competition (and who will be less than 20 years old in July 1988).

For the 20 students, there will be monthly training till December, then fortnightly training beginning January 1988. In February or March, the IMO team will be selected. There will be another training camp, in June 1988, before the team leaves for Australia.