A geometrical figure associated with the Cycle of Sixty Years

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The twelve animals of the Chinese Zodiac are familiar now in the West, but it was not until I read *Spectrum of Chinese Culture* by Lee Siow Mong (Pelanduk Publications (M) Sdn Bhd, 1986) that I learnt about the Cycle of Sixty years. The twelve animals appear five times each in a regular manner in the Cycle, as do the twelve Earthly Branches; the ten Heavenly Stems appear six times each in a regular manner. But the five Elements (Gold, Wood, Water, Fire, Earth) are assigned one to each year in an irregular way, presumably so that a Heavenly Stem will not always be associated with the same Element.

Each Element is always assigned to two consecutive years; years 1 and 2 are *Gold*, 3 and 4 are *Fire*, 5 and 6 are *Wood*, etc. Thus the sequence of Elements, which repeats after thirty years, can be regarded as a sequence of length fifteen; this sequence is as follows:

Gold Fire Wood Earth, Gold Fire Water Earth, Gold Wood Water Earth, Fire Wood Water ,

which seems to have some sort of rhythm even though it is not quite regular. If we view this sequence cyclically, assigning the fifteen terms of the sequence to points equally spaced around a circle, and then join the three points associated with the same Element to form a triangle, we obtain Figure 1, a pleasing pattern which shows that the five Elements are all treated in the same way in the cyclic sequence. Note that the five triangles are obtained from each other by successive rotations through 72°.

Even with this irregularity in the sequence of Elements, each Element is associated with only six of the ten Heavenly Stems during the Cycle of Sixty Years. We cannot change the traditional sequence, but we can ask a mathematical question: "is it possible to obtain a sequence of *thirty* 'double years' for the five Elements, by creating a figure in which thirty points equally spaced around a circle are divided into five hexagons, the hexagons being obtained from each other by successive rotation through

72°, in such a way that each Element is associated at least once with each Heavenly Stem during the Cycle (and thus any given Element will be associated twice with only two Heavenly Stems)?" The following conditions must be satisfied.

(a) The distance (measured around the circle) between any two points of a hexagon must not be multiple of 6 (otherwise rotation through multiples of 72° would produce two hexagons with a common vertex).

(b) Only one pair of vertices of a hexagon can be separated by a distance that is a multiple of 5 (because this pair of vertices will produce an Element that is associated twice with two Heavenly Stems during the Cycle).

A desirable third condition is that the vertices of each hexagon should be well spaced out around the circle.

Before considering how to obtain such hexagons, let us examine Figure 1 more closely. Figure 2, like Figure 1, contains five triangles in a circle, and the vertices have been numbered. Let us "multiply the vertices of each triangle by 4, modulo 15"; this means that, for instance, we take the vertices 5, 6, 7 of one triangle, multiply by 4 to give 20, 24, 28, then divide these numbers by 15 and take the remainders, namely 5, 9, 13, as the vertices of a new triangle. The five new triangles obtained in this way are the triangles of Figure 1. The number 4 can be used as a multiplier because it is coprime to 15 (this means that the highest common factor of 4 and 15 is 1). If we multiply the triangles of Figure 2 by 2 instead of 4, we obtain Figure 3, which gives another perfectly good sequence of Elements except that the Elements are not so well spaced out around the circle. The reader can easily obtain the triangles produced by multiplication by 7, and will also discover that multiplication by 8, 11 and 13 yields the same patterns of triangles as before.

We can return now to the problem of the hexagons. Figures 4 and 5 show two patterns of hexagons satisfying conditions (a) and (b), but the vertices are not well spaced around the circles (the circles themselves have been omitted in these two figures because they obscure the shapes of the hexagons). If we multiply the vertices of Figure 4 by 13 we obtain Figure 6. If we multiply the vertices of Figure 5 by 11 we obtain Figure 7. Note that the hexagons are just a convenient way of showing which six points on the circle are associated with the same Element; these points can be joined in any suitable order, and in Figure 7 each point is joined to its nearest neighbours whereas in Figure 6 the points are joined as they were before being multiplied. Of the two figures, Figure 6 provides a better spacing of Elements around the circle.

Whatever way is chosen for joining up the vertices of the hexagons, we do not get a figure with such elegant simplicity as Figure 1. The reader may like to pursue these investigations further, by trying to use asymmetrical triangles and hexagons for instance, but Figure 6 seems to provide the best possible spacing around the circle

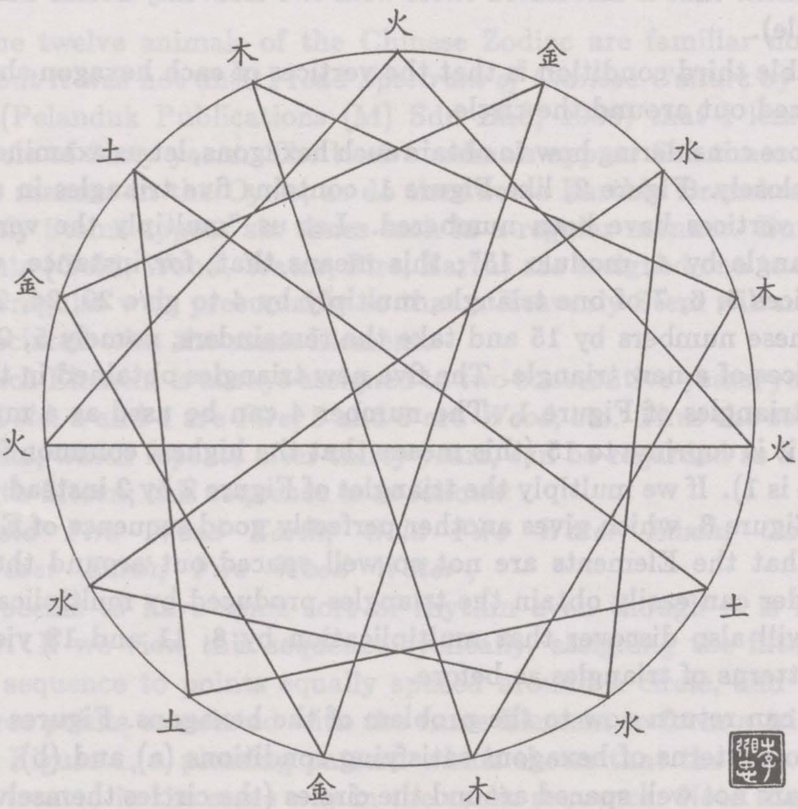

Figure 1

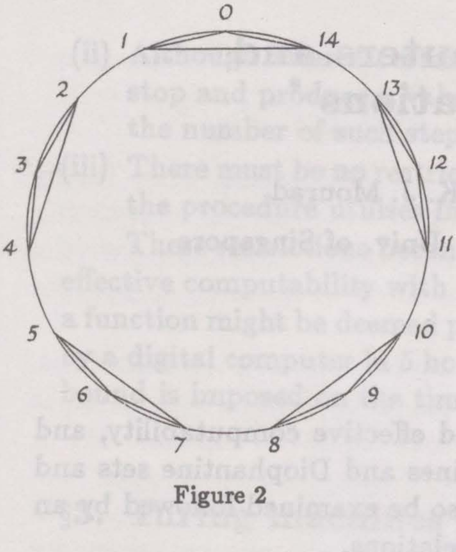

Figure 2

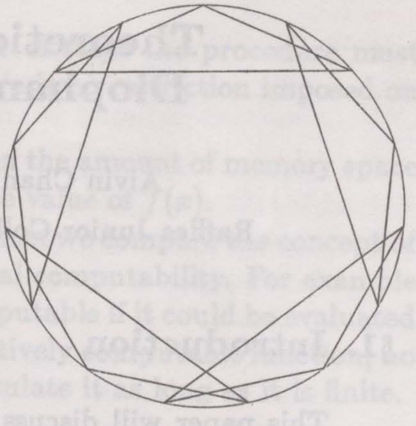
Figure 3

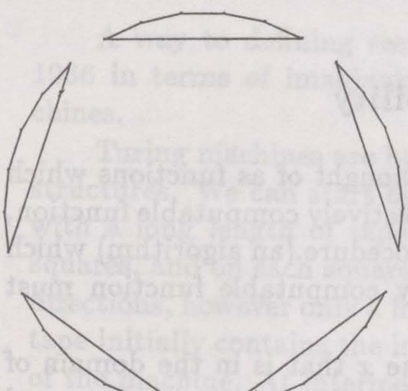
Figure 4

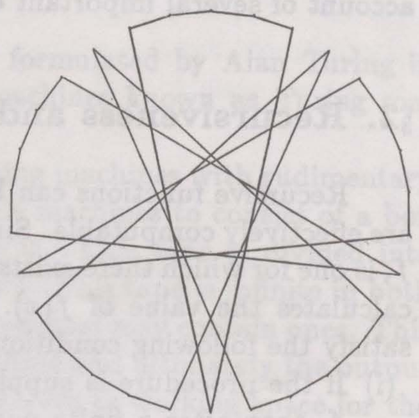
Figure 5

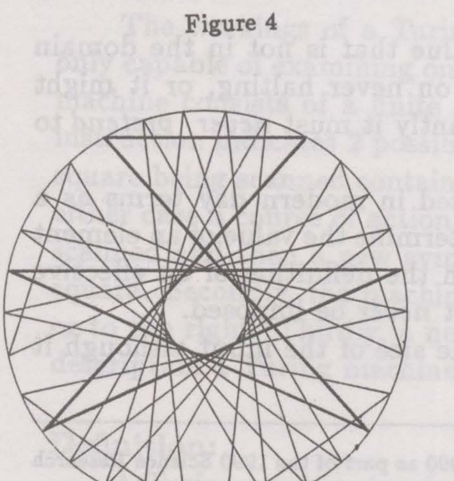
Figure 6

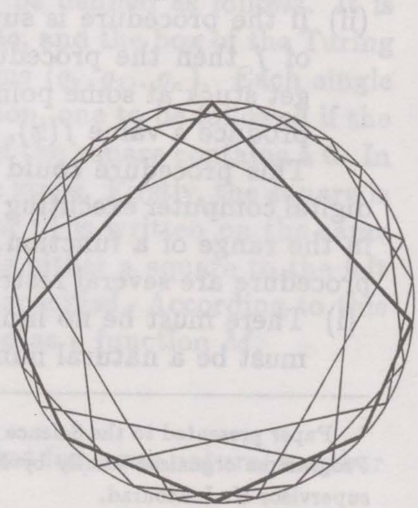
Figure 7