Chaos and Fractals

R. B. Potts
Visiting Professor, Department of Mathematics
National University of Singapore

Summary of a talk given to the Singapore Mathematical Society Student Prize giving ceremony, Friday 13 September, 1991.

The basic concept of the new science of Chaos is a mathematical one. (Note the use of a capital letter to indicate that Chaos has a special meaning different from the word chaos used in everyday language.) The 'logo' of this new science is the Mandelbrot BEETLE. This can be drawn on a rectangular grid following relatively simple arithmetical rules.

The procedure can be illustrated by a game of BEETLES, played either as singles (one dimension) or more interestingly as doubles (two dimensions). The rules of the game are given below.

A computer can be easily programmed to play the game using for the grid a video screen consisting, say, of 300 $\times$ 200 pixels. For rule 5 of the game, the number of iterations might be increased, say, to 100. With a colour monitor, the BEETLE can be produced in fascinating and beautiful complexity by assigning to each pixel a colour corresponding to the number of iterations recorded from rule 4.

The boundary of the BEETLE forms a fractal of dimension between 1 and 2. Its intricate structure can be revealed by zooming in on a small region near a part of the boundary. The zoom is achieved by simply making the calculations more accurate and using more significant figures. The so-called Order within Chaos is indicated by the repeated appearance of the BEETLE, slightly modified, as the power of the zoom is increased.

Chaos and fractals have to be seen to be appreciated, and the talk was illustrated with a computer package FRACTINT run on PC's.