WHY "BODMAS"?

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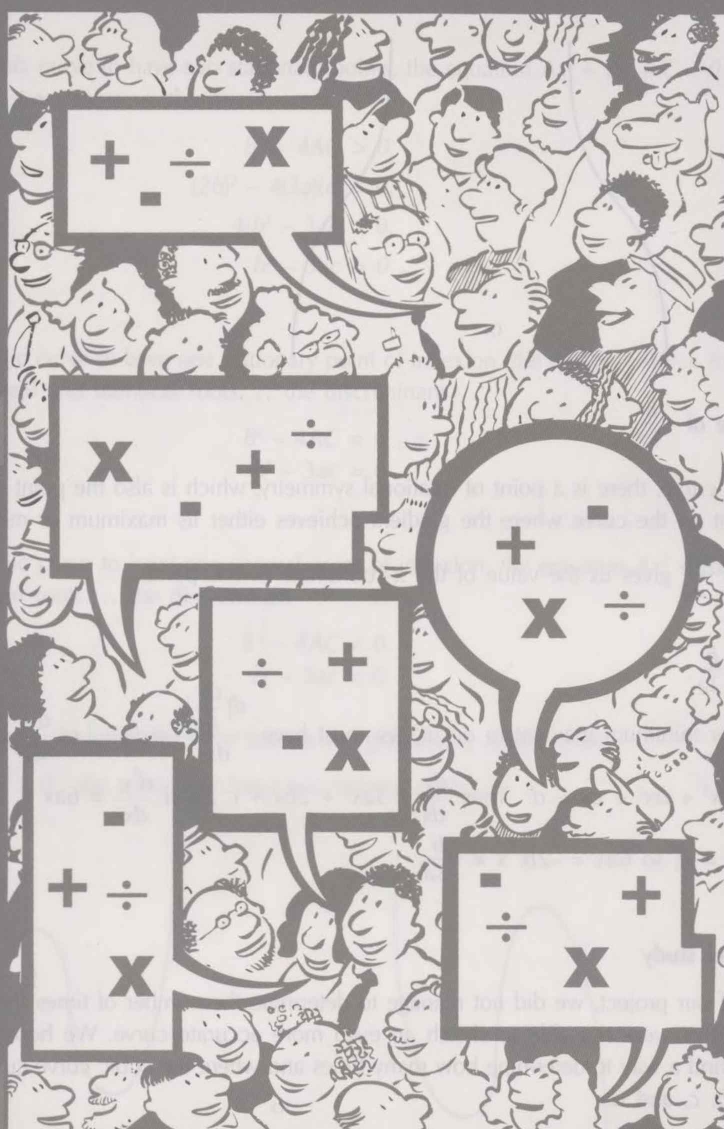

Introduction

All students have come across the 'BODMAS' rule, the rule which specifies the order of the four operations ÷, x, + and -, in conjunction with the use of brackets. Below is a conversation between a mathematics teacher (T) and two of his good students (S1 and S2). The content of the conversation gives some insight into the logical and practical reasons for the 'BODMAS' rule.

Why are rules needed?

S1, S2 : Good afternoon, Sir. We would like to ask you about the BODMAS rule which we learned many years ago. Are you free now?

T : Yes, certainly. What is your problem? Do you know how to use the rule?

S1 : Yes, we do; it's just that we would like to know how this rule came about? We are puzzled over its origin. Isn't it possible to compute arithmetic expressions without referring to any rule?

T : Let me answer to your second question first. Look at this picture. (Refer to Figure 1) What do you see?

S1 : Oh, it's a mess, ...

S2 : Traffic jam! ... or accidents?

T : Well, do you all agree that a jam like the one in the picture will occur if there are no common traffic rules or laws?

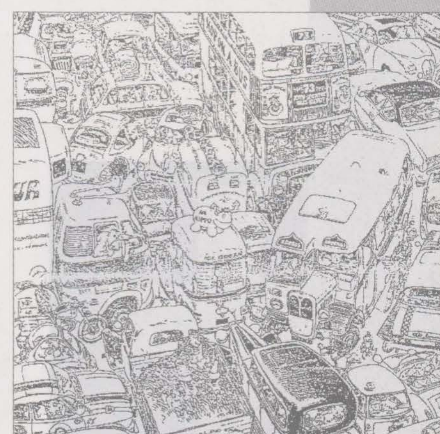

Figure 1

S1, S2 : We suppose so.

T : Can you tell me why?

S1 : Hmm ... . It is clear to us, but it is hard to explain ... . Let me try to illustrate by a simple situation. Suppose, I drive when I see the traffic light turn green, but you drive when the traffic light is red. We follow our own rules strictly, but accidents will still happen ... .

S2 : ... because we do not have a common rule!

T : Good. It's the same situation in arithmetic computation. If there are no common rules for performing an arithmetic computation, we may end up with different answers. It will also be difficult for us to buy or sell things, to mention just one everyday example.

S1 : O.K, we agree that there should be a common rule for the computation of arithmetic sum. But, why BODMAS and not any other rules?

T : Good question. Before we try to rationalise why such a rule is selected, I would like you to tell me what BODMAS is, first.

S1 : (Look at S2) Your turn now.

S2 : Hmm ... BODMAS is the rule of order of the four basic operations, namely, division (÷), multiplication (x), addition (+) and subtraction (-). The rule requires us to deal with terms in brackets first, if there is one, followed by multiplication or division, whichever comes first when reading from left to right, before addition or subtraction, again whichever comes first from left to right.

T : Very good. Now, we must set some criterion so that the execution of the 'preferred' rule is possible. Do you both agree that a rule should be as 'natural' as possible with more advantages than disadvantages?

S1, S2 : (Both smile) Yes, naturally.

T : I would try to show you that, in fact, BODMAS is quite natural. There is no doubt about handling the brackets first (looking at the students for any disagreement or doubt), right?! ... as the purpose of introducing the brackets, in the first place, is to indicate priority in dealing with operations within the brackets first. What is not so clear is the priority of performing mulitplication (or division) over addition (or subtraction).

'The left to right rule'

T : Before we discuss about the priority of one operation over the other, we need to point out a rule which we have always unconsciously applied without realising its existence: the left to right rule. It says that when an arithmetic expression involves only addition (or subtraction) or only multiplication (or division), we compute 'from left to right'.

S1 : Oh, yes. We always do it that way.

S2 : Yeah. For example, $9 \div 3 \div 3$ equals 1 and not $\frac{1}{9}$.

T : Yes, you find this rule well accepted because the rule allows us to compute naturally. It is consistent with the way we read in the English Language, which requires us to read from left to right. Right?

S1, S2 : Yes!

Priority of multiplication over addition

T : Next, let us figure out why multiplication should be performed before addition. Don't forget that a rule is accepted only when people find it natural and it has more good points than bad ones. Let us recall the meaning of multiplication first. Do you agree that multiplication was 'invented' to represent repeated addition?

S2 : Sir, what is repeated addition?

T : Oh, let me give you an example. Instead of adding 7 seven times over (write $7 + 7 + 7 + 7 + 7 + 7 + 7$ on a piece of paper), we denote this (point at the terms written) by 7×7. It is this very purpose of the definition of multiplication which necessitates the rule of performing multiplication before addition. Let me illustrate the necessity of the rule by using an example. Suppose we like to know how much we should pay for a loaf of bread priced at \$1.50 and three cans of soft drinks priced at \$0.70 per can, what will we do?

S1 : It's easy! It should be three times seventy cents which gives me two dollars and ten cents, and then add on to the price of bread, one fifty, ... Mmm, it should be three dollars and sixty cents. Right?

S2 : Yeah!

T : Good. Do you notice that you have done the multiplication first before you add the price of the bread (write $0.70 \times 3 + 1.50$ on the paper)? You are doing it naturally without realising it. Notice that in this natural setting, multiplication is performed before addition!

S2 : O.K.. It is natural to do multiplication first, before the addition, but does it have any other advantages?

T : Yes. In fact, in any practial situation which requires sorting, we would normally group 'like' items together. This natural tendency is also illustrated by the way our number system is built upon the idea of place values. For example, the numeral 123 denotes one hundred, two tens and three ones. The total sum of 1×100, 2×10 and 3×1 gives us precisely the value represented by 123. (Write $1 \times 100 + 2 \times 10 + 3 \times 1 = 123$ on the paper). Notice that in this instance, multiplication is performed before addition too... Next, let us look at two simple arithmetic expressions:

(write $1 + 2 \times 3$ and $2 \times 3 + 1$ on the paper).

It's easy to see that both expressions will give identical values of 7 if multiplication is performed before addition. Suppose we perform otherwise, that is, addition first before multiplication. The expression 1 + 2 x 3 will give 9 and the expression 2 x 3 + 1 will give 8. Both expressions give different values. It means that if we adopt the rule of performing the addition before the multiplication, we may have to be very careful with the way we write our expression. We will have to face inconvenicences, especially in solving word problems.

S1 : Hmm... now I am convinced why multiplication is done before addition... What about division and subtraction?

Division and subtraction

T : A good question, indeed. In fact, division is closely related to multiplication and subtraction is closely related to addition. Let me illustrate this to you. (He writes down $n \div m = n \times (\frac{1}{m})$ and $n - m = n + (-m)$.) You see, we can change the operation of division to the operation of multiplication and vice versa. Similarly, the operation of subtraction can be changed to addition easily, and vice versa. Thus, multiplication and division enjoy equal 'status' from the point of view of order of computation. Also, addition and subtraction enjoy equal 'status' too. Therefore, the need to perform division before subtraction or addition follows logically from the rule of performing multiplication before addition. And, if an arithmetic sum involves only mulitplication and division, we will follow the rule of doing it from left to right and not bother about whether a division or multiplication is performed first.

S1, S2 : Thank you, Sir. We are now clear about the rule of BODMAS.

T : Actually, the argument we have gone through are by no means universal. Rather, they are just products of my own inquiry and construction while engaging myself in reflection on the rule of BODMAS. I would like you to probe further and discover more convincing reasons for the rule of BODMAS. If possible, find out any inconsistencies which may be inherent in such a rule. By the way, have you ever thought of multiplying two whole numers from left to right?

S1, S2 : What do you mean, Sir?

T : Well, when we multiply two whole numbers, we usually use the 'right to left' approach which is taught in primary school. Take, for example, 3456 x 2. We multiply 6 by 2 first before moving on to 5 by 2, and so on, and in between the multilplication process, we add 1 (carry over 1) to the next place value if a product is 10 or more. Get it?

S1, S2 : Yes.

T : But, have you ever wondered whether it would be possible to perform multiplication 'from left to right', which is consistent with the way we read, rather than the usual 'right to left' approach?......

CAN YOU THINK OF A WAY TO DO IT? M²

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