Mathsemantics

by Yan Kow Cheong

"*Without language, math is for the birds.*" Edward MacNeal

Mathsemantics, the science of the semantics (Semantics is defined as "the study of meanings.") of mathematics owes much to Edward MacNeal who points out that to make sense of numbers, we need to command both mathematics and their semantics. Note that the word "mathsemantics" contains every letter in "mathematics" in the same sequence, in addition to the letters *s* and *n*: Math***s***ema***n***tics.

MacNeal remarks that a lot of people who can use words or do math reasonably well can't do both well at the same time. We need to look for mathematical meanings in action or the semantics of numbers. Traditional schooling mostly fails to solve the semantic problems involved in applying numbers to events. That perhaps explains why many adults fear to quantify or fumble when they do attempt to quantify. Even those who have taken advanced mathematics classes exhibit mathsemantic difficulties, for example, many calculus students fear "word" or "story" problems more than other types of problems.

Let us look at some combined-math-and-semantics problems (the answers are given at the end of the article):

1. Solve the following additions.

(a)
$$\begin{array}{r} 2 \text{ apples} \\ \underline{+\ 3 \text{ orange}} \end{array}$$

(b)
$$\begin{array}{r} 5 \text{ managers} \\ \underline{+\ 3 \text{ clerks}} \end{array}$$

(c)
$$\begin{array}{r} 5 \text{ one-way trips} \\ \underline{+\ 3 \text{ round trips}} \\ \text{round trips} \end{array}$$

2. Solve the following multiplications.

(a)
$$\begin{array}{r} 8 \text{ workers} \\ \underline{\text{x}\ 1.5 \text{ hours}} \end{array}$$

(b)
$$\begin{array}{r} 12 \text{ cars} \\ \underline{\text{x}\ 1.5 \text{ miles}} \end{array}$$

3. Solve the following division.

$$30 \text{ min.} \overline{\big)\ 4 \text{ hrs. } 18 \text{ min.}}$$

At first glance these above math-and-semantics problems seem meaningless. We can't add apples and oranges; only numbers can be added, not things. That is logical, but impractical and operationally false. Even when we claim that we can add one apple to another apple, we behave as if the two items are exact duplicates of each other.

No two apples are the same. In fact, no two anything are the same. Following that line of reasoning, we can't add anything. If no two things are alike, what are the rules for adding things? We can't divorce arithmetic and semantics. The question is not whether we can add different things, but how we can add them in clear and useful ways. That leads us into meanings - into semantics.

Attempts to answer these mathsemantic problems often yield answers which contain either "Good math, but bad semantics", or "Good semantics, but bad math". They are mathematically right, but semantically wrong, or vice versa.

1 hr. 34 min. 3 hrs. 45 min. + 4 hrs. 12 min. 8 hrs. 91 min.	1 hr. 34 min. 3 hrs. 45 min. + 4 hrs. 12 min. 9 hrs. 30 min.	1 hr. 34 min. 3 hrs. 45 min. + 4 hrs. 12 min. 9 hrs. 31 min.
Good math, bad semantics.	*Good semantics, bad math.*	*Good math, Good semantics.*

Even applying the simplest mathematical models becomes a question of cultural categorisations. For example, we say that "2 apples + 2 pears = 4 fruits", but "2 pants + 2 jackets = 2 suits." A generation ago, we might have said that "2 men + 2 women = 2 couples", but now we are careful to say that "2 men + 2 women = 4 people".

Whenever we add *things*, we necessarily add *different* things, which we must then group under the same *name*. For example, in the "sheep-and-cow mathematics" or "the mathematics of things", we have the following:

> **One sheep + one sheep = Two sheep**
>
> **One sheep + one cow = Two animals**

In the multiplication "8 workers x 1.5 hours", we should be aware of the difference between a worker-hour and an hour worked. A *worker-hour* is the duration of one worker for one hour, which is timeless, while an *hour worked* refers to a completed event. In other words, the differentiation is about *worker-hours* versus *hours worked.*

Percentage

Let us consider some practical examples involving percentages, which can surprise the unprepared mind.

> ***If sales drop by 20% and then grow by 25%, are you better off?***
>
> You are back where you started!

> ***A drop of 70% followed by a rebounded of 80% leaves one in deep soup***
>
> Example: \$100 - (70% of \$100) is \$30
> \$30 + (80% of \$30) is \$54

> ***A man sold 10 watches and made a total of 20% profit. What was his profit percent for one watch?***
>
> Not 2%, but 20%. Can you see why?

> ***A gain of 80% followed by a loss of 70% is equally bad.***
>
> Example: \$100 + (80% of \$100) is \$180
> \$180 - (70% of \$100) is \$54

In any fluctuation, the percent going up has a smaller base than the percent coming down. Therefore the percentage gain must be larger then the percentage loss, sometimes much larger, just to stay even. When we ask, "What is the percent increase?", we are really asking, "What percent of the base amount is the increase?"

> ***A national survey shows that 48% of teenage boys and 32% of tennage girls are cigarete smokers. This may lead to the newspaper headline, "Eight out of ten teenagers smoke."***

It is easy to see that the wrong conclusion was derived by simply adding the two percentages. A reasonable computation, assuming equal numbers of boys and girls, tells us that the percent for a combined base must lie between the percents for the separate bases. Thus we don't add 48% and 32%, but take the average, 40%.

The pure mathematics of percents may seem easy, but their mathsemantic uses are hard. The mathsemanticist Edward MacNeal was right when he said, "Percentages are dangerous social and economic tools that appear easy only to math teachers and the inexperienced."

The Speed Problem

> ***A car travels from A to B at a speed of 40 km/h and then returns from B to A at a speed of 60 km/h. What is the average speed for the entire trip?***

This classic speed problem is often calculated incorrectly because of a numerical prejudice most people have about the concept of average. An average is found by adding two or more numbers, and dividing the sum by the number of numbers that are added. Even though we can still use this method here, it can only be correct if we use various speeds for every instant of time.

The concept of average speed requires that we take the total distance divided by the total time taken to travel that distance. In this problem, we are not given any distance. Let us look at the solution.

Note that 48 km/h is not the same result as taking 40km/h plus 60 km/h, and dividing by 2, which would give 50 km/h, an incorrect answer.

This simple example shows that even when dealing with simple fundamental concepts such as the average speed, we can easily fail to grasp its desired meaning. How much more so with more complex problems involving quantities derived from this concept, such as velocity, kinetic energy, power, and so on?

Conclusion

That we are ill-prepared to handle problems involving both mathematics and semantics should make us aware that we often lack the mathsemantic sophistication necessary to judge newspapers' stories or advertisements promoting sales, especially those involving percents, big numbers and statistics. As a result, we are prone to number abuse by both mathsemantically-challenged reporters and unscrupulous writers.

Until there is a law against mathsemantic pollution, we are at the mercy of mathematical charlatans and New Age gurus who are preying on the innumerate minds of the public.

Answers:

1. (a) 5 fruit, or five pieces of fruit, or the like.

   (b) 8 staff, or the like.

   (c) $5\frac{1}{2}$, or 5.5.

2. (a) 12 worker-hours, or 12.0 wk-hrs, or the like.

   (b) 18 car-miles.

3. 8.6, or $8\frac{3}{5}$.

References

MacNeal, E. **Mathsemantics : making numbers talk sense**, Viking Penguin. (1994)

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