Problems ? Corner

Problem 1

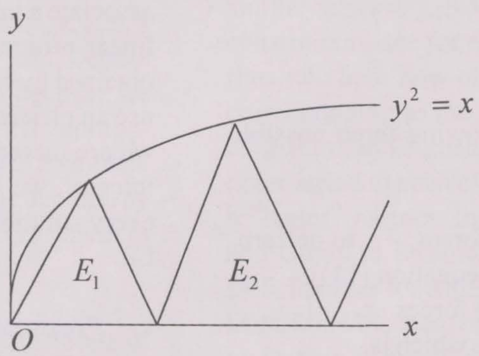
If n is a positive integer and $1+3^n+9^n$ is a prime number, prove that there exists a nonnegative integer k such that $n=3^k$.

Prize

One $100 book voucher

Problem 2

Let S denote the region which is bounded below by the x-axis and bounded above by the parabola $y^2 = x$. A sequence of equilateral triangles $E_1, E_2, \ldots$ is constructed in S starting from the origin O as shown in the diagram. Find the perimeter of E_{1998}.



Prize

One $100 book voucher

CONTEST

1. *Prizes in the form of book vouchers will be awarded to the first received best solution(s) submitted by secondary school or junior college students in Singapore for each of these problems.*

2. *To qualify, secondary school or junior college students must include their full name, home address, telephone number, the name of their school and the class they are in, together with their solutions.*

3. *Solutions should be typed and sent to :*
   The Editor,
   Mathematics Medley,
   c/o Department of Mathematics,
   National University of Singapore,
   Kent Ridge, Singapore 119260;

and should arrive before 30 April 1999.

4. *The Editor's decision will be final and no correspondence will be entertained.*

Solutions

to the problems in volume 24 No. 2 September 1997

Problem 3 (Prize : One \$100 book voucher)

Let $[x]$ denote the largest integer which does not exceed x.

For example $[3.21]=3$.

For each positive integer n, define $a_n = n + \left[\dfrac{1997}{n}\right]$.

Find the smallest number in the sequence $\{a_1, a_2, a_3, \cdots\}$.

Solution to Problem 3
by Tadinada Karthik, VJC, Class 97S36.

The answer is 89.

Let $\{x\}$ denote the fractional part of the number x.

Since $n+\dfrac{1997}{n} = n + \left[\dfrac{1997}{n}\right] + \left\{\dfrac{1997}{n}\right\}$

and $n+\dfrac{1997}{n} = \left[n+\dfrac{1997}{n}\right] + \left\{n+\dfrac{997}{n}\right\} = \left[n+\dfrac{1997}{n}\right] + \left\{\dfrac{1997}{n}\right\}$.

Therefore $a_n = n + \left[\dfrac{1997}{n}\right] = \left[n+\dfrac{1997}{n}\right] \geq \left[2\sqrt{1997}\right] = 89$

by the Arithmetic mean-Geometric mean inequality. Since $a_{45}=89$, therefore this lower bound is attained and hence is the minimum.

Solved also by V. Srividhya, YJC, Class 1S5; Shiau Vee Hueng, RJC, class 1A01D; Chong Chin Yuan, RJC, Class 2S06B and Chia Meng Hwee, RJC, Class 2S06B. One incomplete solution was received.

Editor's Note:
The prize money was shared equally between Tadinada Karthik and V. Srividhya.

Problem 4 (Prize : One \$100 book voucher)

There are 2998 points inside a circle which has area 1 unit. Prove that it is possible to choose three points among them such that the triangle formed by using these three points as vertices has an area less than

$$\frac{1}{1998}.$$

Solution to Porblem 4
by Tadinada Karthik, VJC, Class 97S36.

Divide the circle into 999 equal sectors.
Since $3 \times 999 = 2997 < 2998$, therefore by the Pigeon Hole Principle at least one sector has 4 points in it. Call these four points A, B, C and D.

Divide the quadrilateral $ABCD$ into two non-overlapping triangles, say ABC and ABD. We may assume that area of triangle $ABC \leq$ area of triangle ABD.
Then area of triangle $ABC \leq \dfrac{1}{2}$ area of quadrilateral $ABCD$

$$< \frac{1}{2}\left(\frac{1}{999}\right) = \frac{1}{1998}.$$

One incomplete solution was received.

Editor's Note:
The prize money went to Tadinada Karthik.