I want to be a Number Theorist

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Mathematics has always been my favorite subject in schools. In the beginning, I like mathematics because I can solve problems which my teachers assigned in class. The turning point came when I was in Secondary 3. I had a classmate, Lim Yee Wei, who happened to know a lot of interesting facts about mathematics and encouraged me to read Martin Gardner's "New mathematical diversions from Scientific American". It was in this book that I found many mathematical games and recreational problems that I had never seen before. I liked Gardner's book so much that I asked Yee Wei for more recommendations. With the permission of his father, Mr Lim Kim Woon, Yee Wei began to lend me books over a period of two years. It was during this time that I learnt about prime numbers, transcendental numbers π and e, continued fractions and diophantine equations. One day while reading an article about the famous Indian mathematician Srinivasa Ramanujan[1] by J.R. Newman[2], I saw the continued fraction

$$\cfrac{e^{-2\pi/5}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\dots}}}}=\sqrt{\frac{\sqrt{5}+5}{2}}-\frac{\sqrt{5}+1}{2}.$$

Having learnt about e, (, continued fractions and algebraic numbers, I find the identity truly amazing! Identity (1) is contained in Ramanujan's first letter to G.H.Hardy, a famous number theorist, who admitted that he was totally defeated by this identity. The beauty of identity (1), as well as Hardy's "challenging remark", strongly motivated me to seek a proof of (1). After several serious attempts, I failed. Then, because of the GCE 'A' level examination and the 2.5 years of national service, I had to postpone my little project.

In 1988, I began my first year in the National University of Singapore. In the excellent Science Library, I found the book "Pi and the AGM" by J.M. Borwein and P.B. Borwein. There, the continued fraction was mentioned again, together with references to its proofs. I then began a small survey on (1), organising results from the classic "Introduction to Theory of Numbers" by G.H. Hardy and E.M. Wright, and research papers by G.N. Watson and K.G. Ramanathan. This was the first time I read research papers. By now, I had already encountered elliptic functions, Rogers-Ramanujan identities and partitions. This survey was completed at the end of my first year.

In 1990, the Department of Mathematics of NUS invited the famous Fields Medalist Prof. J.P. Serre to conduct a short course in Number Theory. I was very fortunate to know about it (from Dr. Thomas Bier) and attended the whole course. At the suggestion of Dr. Tan Eng Chye, I showed Prof. Serre my little survey. After reading it, he said, "Nice." I was truly motivated by this simple comment and eventually, I wrote up two more surveys, one on Ramanujan's proof of the Bertrand's Postulate (which says that there always exist a prime p between n and 2n for any integer), and the other one on Ramanujan's famous partition identity

$$\sum_{k=0}^{\infty} p(5n+5)q^n = 5\prod_{k=1}^{\infty}\frac{\left(1-q^{5k}\right)^5}{\left(1-q^k\right)^6},$$

[1] For an excellent account of the life of S. Ramanujan, see "The man who knew infinity" by R. Kanigel.
[2] The article is in the book "The world of Mathematics", Vol. 1, edited by James R. Newman.

where $p(n)$ is the number of unrestricted partitions of n.

During my third year, my current colleague and friend, Dr. Lang Mong Lung, encouraged me to apply for a teaching assistant position in U.S. and pursue my Ph.D. degree at the same time.
So, I wrote to Prof. Serre for advice, telling him that I would like to work in Number Theory. Serre indicated that since I was interested in the work of Ramanujan, I should apply to either University of Illinois at Urbana Champaign (B.C. Berndt), or Pennsylvania State University (G.E. Andrews) or University of Wisconsin (R. Askey). It was a tough decision to make as they are all excellent mathematicians. While trying to ask for a recommendation letter from Prof. D.J.S. Robinson, a visiting professor who taught me algebra, he told me that he had a colleague who worked on editing Ramanujan's notebooks and his name was B.C. Berndt! The choice for me then was very clear and eventually I applied to University of Illinois, with recommendations from Prof. Serre, Prof. Robinson, Prof. J.Berrick and Dr. Cheng Kai Nah. I obtained an assistantship and began my graduate study in 1991 under the supervision of Prof. B.C. Berndt. In 1992, I wrote my first paper, giving a new and elementary proof of (2). This shows that my hard work paid off. Then together with my advisor and L.C.Zhang, we obtain new results on continued fractions, from which (1) is a special case.

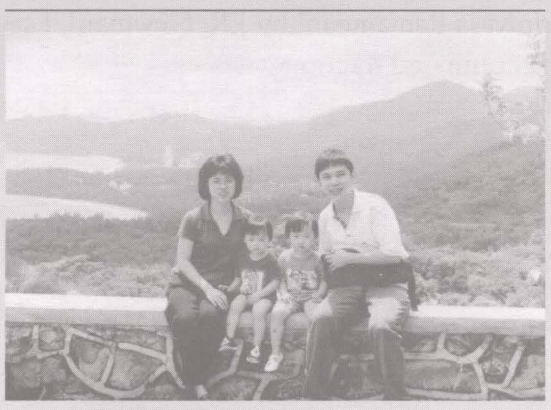
In 1995, I graduated and spent nine months at the Institute for Advanced Study in Princeton. The job market in 1996 was very bad everywhere and I was only offered a one year visiting position in National Chung Cheng University, Taiwan. I accepted the offer and the experience was great. I returned to the National University of Singapore in 1997 as a tenure-track assistant professor. In 1999, I was awarded the Young Scientist Award from the Singapore National Academy of Science (This is the reason why I have been asked to write this article).

Finally, here is a joke that I like to tell using one of my own identities. Let

$$G(q) = \cfrac{q^{1/3}}{1+\cfrac{q+q^2}{1+\cfrac{q^2+q^4}{1+\cfrac{q^3+q^6}{1+\ldots}}}}, \quad |q|<1.$$

What is the value of $\left(G\left(e^{-\sqrt{2\pi/3}}\right)\right)^3$? If you answer $G\left(e^{-\sqrt{2\pi}}\right)$ (by accidentally placing the '3' into the exponent), you get partial credit because the answer is

$$\left(G\left(e^{-\sqrt{2\pi/3}}\right)\right)^3 = \frac{G\left(e^{-\sqrt{2\pi}}\right)}{2}.$$



Chan Heng Huat attended Pei Chun Primary School, The Chinese High School and National Junior College. He graduated from the National University of Singapore in 1991, and obtained his Ph.D from University of Illinois at Urbana Champaign, USA. He is now an Assistant Professor at the National University of Singapore. Heng Huat won a Young Scientist Award in September 1999 for his work in number theory, in which he has derived many exciting identities. The Award is organised by the Singapore National Academy of Science and supported by the National Science & Technology Board.