Celebrating Mathematics

A Cover of the 12th Annual Singapore Mathematics Project Festival 2012

"While one may question the future job perspective of someone reading Mathematics, keeping in touch with the subject provides one with a competitive edge over others as they are capable of utilising the many skills that the subject develops over time – critical thinking and problem-solving being just two of the many." This was the main message delivered by Professor Zhu Chengbo, President of the Singapore Mathematical Society (SMS), in his opening speech at the Singapore Mathematics Project Festival (SMPF) 2012. NUS High School of Mathematics and Science played host to the 12th Annual SMPF, held on the 17th of March 2012, for the very first time since the festival's inception in 2001.

The object of the festival is three-fold – it aims to encourage secondary school students in Singapore to carry out and present innovative and creative work in Mathematics; to recognize and emphasize the importance of project-based learning in the secondary school curriculum and to complement the examination-based Mathematical Olympiads organized by the SMS. The festival is divided into two sections; the Junior Section, primarily comprising of Secondary One and Two students and the Senior Section, targeting more Mathematically-matured students from secondary schools. After competing in the preliminary rounds held on the 11th and 18th of February 2012, five teams were selected for each section to present their projects in the Grand Finals for a chance to win the Foo Kean Pew Memorial Prize (a cash prize of S$1,000 for each section) and clinch the Singapore Mathematical Society Shield for their schools.

This year the festival saw a wide range of Mathematical ideas being explored, ranging from disciplines such as Number Theory, Graph Theory and Geometry. When asked as to why his team had chosen to focus on a Pure Mathematics topic such as Fermat's Last Theorem, Le Minh Phuc mentioned that they were looking through various Mathematical papers when they chanced upon the topic. "We came across this topic and thought to ourselves – what can we do to further this theorem?" added his National Junior College team-mate Tong Hien Chi, claiming that they were impressed by the history of the theorem and how it acted as a "golden egg" in Number Theory as the quest for a proof of the theorem gave rise to various intriguing results. Their project (which was submitted to the Senior section) – titled *The Extension of Fermat's Last Theorem* – introduces one more unknown into the famous equation ($a^n + b^n = c^n$) and investigates whether there exists four positive integers satisfying the new equation ($a^n + b^n + t^n = c^n$) for positive values of n.

Not all the projects presented at the festival were purely for academic pursuits. The students from Hwa Chong Institution (High School) chose to focus on a more real-life problem by studying the "green wave" pattern of traffic light in Bishan Street 22 in their project *Project Traffic Light* (which was submitted to the Junior Section). Using the statistics which they had compiled over time, the students constructed Mathematical models and used linear programming methods to optimise traffic light conditions in order to improve the traffic light system of that particular street. Some other students from NUS High School of Mathematics and Science chose to work on a more light-hearted project which focused on the Mathematics behind the board-game Mastermind in their project *Strategy Behind Mastermind* (which was also submitted to the Junior Section) by coming up with various algorithms to win the game in the shortest number of rounds. These projects, which were presented in a very enlightening manner with the aid of various animations to allow for easy understanding, were among the many projects which showed that Mathematics truly had reaches beyond the classroom and that the applications involving it were sometimes in the most unexpected areas.

While there is no denying that many simplifications had to be done to get a grip of the whole problem that they were undertaking and coming up with the project report – as admitted by the participants themselves – this continual process of inquiry involving the finding, testing and correcting of ideas in itself was an enriching journey for most of our participants. Many of the participants were surprised with just how much they had been able to learn during this process of inquiry, something that they did not expect when they first started out with their projects, with their confidence clearly exhibited throughout their presentations and interview sessions. The judges, lecturers from the National Institute of Education (NIE) and the National University of Singapore (NUS), were thoroughly impressed with not only the enthusiasm shown by these participants, but their ability to articulate their thoughts and ideas clearly during the interview sessions. It is such positive attitudes towards the study of Mathematics displayed by secondary school students such as those who took part in this year's SMPF which add fuel to the thriving Mathematical scene here in Singapore.

The following are ten projects presented at the Festival Congress (Final round)

Junior Section

Project 1: Fit the squares into a square!

By Ang Xue Yin Angelin, William
Mentor: Ms Christina Lee
School: River Valley High School

Abstract

Square is a unique polygon, which qualifies as a rhombus, and also as a rectangle. The interesting feature about squares is that it is rigid and rule abiding. Its sides are of equal lengths and the adjacent sides are perpendicular to each other. This allows squares to be easily tessellated, which explains why square is the popular choice of shape in tiling. Square is a subtle yet effective shape to efficiently tile up a given area.

Due to the equal sides of a square and the similar angle of 90 at each vertex, tessellation of squares may be with minimal fuss as compared to tessellating other shapes. However, the special yet rigid properties of the equal sides and equal angles of a square can be a disadvantage when tiling a given area that is also a square. In view of such special properties of squares, we have thus decided to investigate the tiling of squares using smaller squares. When using different sizes of squares to completely tile up a given larger square area, with the minimum possible number of squares used, will it still be convenient to tile all the different dimensions of square areas with the supposedly convenient shape of square? How then can we effectively be able to overcome this problem? This paper will explore the properties of squares used to tile in relation to the square area to be tiled on.

The main objective of this paper is to find the minimum number of squares that are smaller than the square area to be tiled, necessary to cover that square area. The results are then explored for consistent patterns, in search of a solution to find the minimum number of squares needed to tile up any square area.

Project 2: Strategy Behind Mastermind

By Hsiao I Ann, Lee Yue Xin
Mentor: Mr. Chai Ming Huang
School: NUS High School of Mathematics and Science

Abstract

The aim of this project is to find the algorithm to play the Mastermind game so as to break the code within the minimum possible steps, that is, to win the game. For this report, we present the 3 by 3 Mastermind game (3 places available for the code and 3 colors to choose from) with repetition as well as some examples for the 4 by 6 Mastermind game. We tried out different combinations of colour pegs and spaces, and then compare them to see what we could discover from that. We found the number of average guesses and the algorithm to win the game.

Project 3: Project Traffic Light

By Tan Jing Ling, Lee Bing Qian Ryan, Justin Lo Tian Wen
Mentor: Wang Qian
School: Hwa Chong Institution

Abstract

Our project aims to make use of the concept of "Green Wave" to come up with an optimized traffic light formula, in order to improve the traffic light system of a particular street, Bishan Street 22. We carried out our project by compiling actual statistics, and we proceeded to construct mathematical models from them. Finally, we used linear programming methods to optimize traffic light conditions. From our findings, we derived the exact timings in which the pedestrians and vehicles would meet at each traffic light. These allowed us to adjust the timings of traffic lights to provide the most efficient possible scenario to ease the traffic along that street.

Project 4: Napoleon's Theorem

By Tan Jia Yu, Cheryl Chua Xing Jun, Xu Ming Xin
Mentor: Mr Desmond Pang
School: Paya Lebar Methodist Girls' School (Secondary)

Abstract

Napoleon's Theorem states that when equilateral triangles are constructed on the sides of any triangles, either all outward or all inward, the centres of those equilateral triangles themselves form an equilateral triangle. This project seeks to investigate the application of the theorem on regular polygons where regular polygons are constructed on each side of the original polygon. More specifically, it seeks to determine the dimensions of the new polygon formed when the centroids of the constructed regular polygons are connected. By making observations and relevant calculations, this project further attempts to develop a general formula that links the dimensions of this new polygon with the length of the original regular polygon. The team established that the length y, of each side of the resulting regular polygon is related to the length x, of each side of the original regular polygon by the following formula:

$$y = \sqrt{2\left(\frac{x}{\tan\left(\frac{360}{2n}\right)}\right)^2\left(1-\cos\left(\frac{360}{n}\right)\right)}$$

Project 5: Interesting Patterns in the Pascal's Triangle and Pyramid

By Ng Jian Rong, Shashvat Shukla, Glen Goh Wee Zhuan
Mentor: Mr. Chai Ming Huang
School: NUS High School of Mathematics and Science

Abstract

We present the Pascal's triangle along with its patterns and applications in solving some simple math problems. We explain several interesting patterns we observe and extend them to a higher dimension. Pascal's triangle is a triangle made of binomial coefficients in rows. The Pascal's triangle can be used like a mathematician's periodic table. It contains all sorts of wonderful patterns. The powers of 2 and 11, Fibonacci numbers and binomial coefficients are just a few of the things that can be derived from the Pascal's triangle. The triangle can also help us solve some problems, derive formulas and understand math better. It also keeps popping up in various forms and in unexpected places. Now imagine if we were to extend the triangle in a clever way such that it is a "Pascal's pyramid", we would be able to solve and understand a larger variety of problems. Then we could add an extra dimension to that to get a Pascal's 4-dimensional simplex and so on to get a clearer understanding and maybe even generalize some of the patterns and proofs.

Senior Section

Project 1: A General Method of Flattening Convex Prismatoid into Flat Sheet

By Zhou Jingqi, Li Chenglei
Mentor: Mr Cheong Kang Hao
School: NUS High School of Mathematics and Science

Abstract

Prismatoids are polyhedrons where all vertices lie in two parallel planes. In this project, we will only focus on convex prismatoids and will tackle more complex situations with concave prismatoids or even biplanars in future work. We have divided all prismatoids into 5 broad categories and then created one general method for each category to flatten convex prismatoids. Given aerial view and height of a prismatoid, all angles on all faces and all lengths of edges can be found. By following the algorithms we have derived, the crease pattern for flattening can be generated.

Project 2: Weak Roman Domination

By Xu Linfeng
Mentor: Prof Koh Khee Meng
School: Hwa Chong Institution

Abstract

This project focuses on the research of weak Roman domination number. Weak Roman domination is a concept proposed in 2002, which is a new strategy in response to the famous Roman domination problem. In graph theoretic terminology, let $G = (V,E)$ be a graph and f be a function $f : V \to \{0, 1, 2\}$; a vertex u with $f(u) = 0$ is said to be undefended with respect to f if it is not adjacent to a vertex with positive weight. The function f is a weak Roman dominating function (WRDF) if each vertex u with $f(u) = 0$ is adjacent to a vertex v with $f(v) > 0$ such that the function f': $V \to \{0, 1, 2\}$ with $f'(u) = 1, f'(v) = f(v) - 1, f'(w) = f(w)$ if $w \in V - \{u, v\}$ has no undefined vertex. The weight of f is $w(f) = \sum_{v \in V} f(v)$. The Weak Roman Domination number, denoted by $\gamma r(G)$, is the minimum weight of a WRDF in G.
In this project, successful approaches have been made to show that Weak Roman dominating index $r(xy)$ is between 0 and 1, with all the possible cases fully discussed.

Furthermore, the paper shows $\gamma_r(G) \le \left\lfloor \frac{2n}{3} \right\rfloor$ for any connected graph of order n and $\gamma_r(G) \le \left\lfloor \frac{2n-2}{3} \right\rfloor$ for unicyclic graphs. Those upper bounds are crucial when applying the theory into real-life scenarios. This paper also characterizes graphs for those upper bounds to be achieved. For further research, I aim to compare the relative advantages and disadvantages of Roman domination number, weak Roman domination number and domination number, research more on various types of graphs, and link them to real-life applications.

Project 3: Graph Theory: List Assignment Problems

By Lee Kian Wee, Yeo Yao Rui
Mentor: Chia Vui Leong
School: NUS High School of Mathematics and Science

Abstract

The field of list colouring in graph theory is a fairly recent development in mathematics, which has applications in practical problems concerning channel and frequency assignments, as well as scheduling and organization algorithms. This research aims to determine the uniqueness of $(k-1)$-list assignment of a graph G given that the choice number of G is k. In this project, we firstly address 2 questions on a complete bipartite graph $G \cong K_{n,m}$: finding a $(k-1)$-list assignment of G such that it is not colourable given that $ch(G) = k$, and finding a general form of it if this list assignment is not unique. We have established an efficient method "$\otimes$ product" to determine if a k-list assignment for a graph is colourable. With this, we generalise our results on complete multipartite graphs of the form $K_{a*b,c}$.

Project 4: Catalan Numbers: An Attempt to Generalize

By Jonathan Ang Yun Hao, Eun Jung Min, Lee Yi Min
Mentor: Mr Loo Chee Wee
School: NUS High School of Mathematics and Science

Abstract

Last year we did a project titled Catalan Numbers: $m \times n$ and $l \times m \times n$. We explored how the Catalan numbers is able to solve a series of counting problems ranging from Computer Science to Geometry. In this project we attempt to carry on where we left off last year. We seek to generalize the formula for Catalan Numbers with dimensions as the parameter. Though we were not able to prove the general n-dimensional case, we managed to extend the formula we have from the 3-dimensional case to a 7-dimensional setting.

Project 5: The Extension of Fermat's Last Theorem

By Tong Hien Chi, Le Minh Phuc
Mentor: Ms Lye Wai Leng
School: National Junior College

Abstract

Inspired by the famous Fermat's last theorem (no three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than two), this project aims to explore the challenging issue in number theory by introducing one more unknown into the equation. We aim to investigate if there exists four positive integers x, y, z and t that satisfy the equation for positive integer value of n.

In this project, we have proven that for $n = 1, 2$, there exist infinitely many solutions to the equation. We have also introduced a geometric method to find all the solutions for the case of $n = 2$. For $n = 3$, we show one way to generate infinitely many solutions but this method does not give all the solutions of the equation. The same method is applied for $n = 4$ but the result is not as complete.

The problem is potentially another puzzling area of number theory and requires higher-level Mathematics to be solved completely. Elliptic curve or computational methods should also be utilized to solve for cases with large values of n. Another way to generalize this question is to introduce more unknowns and investigate the general solution for an arbitrary number of unknowns.