Bar Model Method for Age Problems

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Abstract
This article shows how Bar Modelling can be used to depict the temporal situations in "age problems". Several examples will be given to illustrate the bar model method. They illustrate the flexibility of the bar model construction for problems of different nature.

Yesterday is but today's memory and tomorrow is today's dream
- Khalil Gilbran (1883 - 1931), Lebanese American artist, poet and writer

1. Introduction

In Singapore Math, two basic models, *part-whole model* and *comparison model,* help in the visualisation of the problem solving process for word problems (see [1]). In our earlier work([2],[3],[4]), bar models with identical bars are used for situations involving ratios and fractions and these models are called *simple bar models.* These simple models enable us to use the so-called *unitary method* to solve problems. In this article, we will use the bar model approach to solve some variants of age problems at primary Olympiad level (see [5], [6]).

Let us recall the bar model method with a simple age problem.

Example 1

The total age of a father and his two sons is now 75. Nineteen years later, the age of the father is $\frac{5}{6}$ of the total age of his two sons. What is his age now?

Solution

In Figure 1, we construct a part-whole model for the total age of the two sons and a part-whole model for the age of the father 19 years from now, noting that the total age of the two sons will increase by $2\times19 = 38$ and the age of father will increase by 19.

A simple bar model (called a fraction bar model) with 6 equal parts is constructed to show that the age of the father is $\frac{5}{6}$ of the total age of the two sons after 19 years. Let each equal part be a unit U as shown.

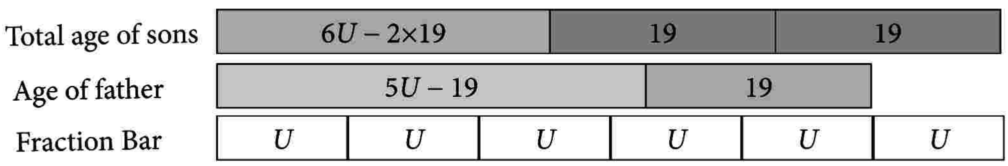

Figure 1

Since the total age of the father and sons is now 75 , we have

$$6U - 2\times19 + 5U - 19 = 75$$

and so $U = 12$.

Therefore the present age of the father is $5U - 19 = 5\times12 - 19 = 41$.

Bar Models for Ages in the Past, Present and Future

Age problems involve past, present (now) and future situations. The key feature of age problems is that

the age of each person increases at the same rate
and so
the age difference between any two specific persons is always constant.

Figure 2 shows the *age difference* as the key feature in a general bar model for two persons A and B whose present ages are a and b (with $b > a$) respectively.

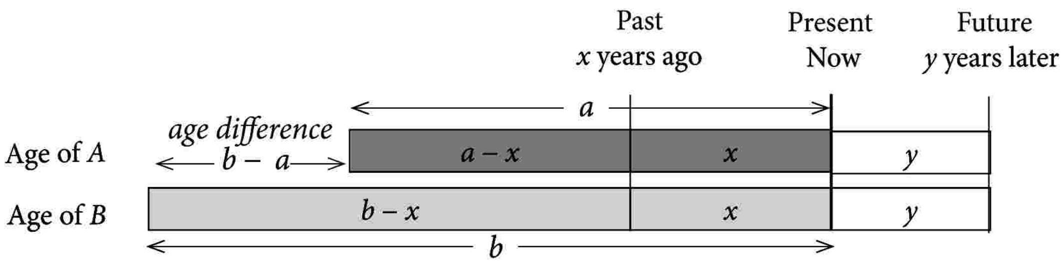

Figure 2

In Example 1, the usual bar models are constructed based on the magnitudes of the ages. In the following examples, we will align the bar models corresponding to the ages at the desired year as shown. It will help to construct appropriate models for the mathematical formulations.

Now, we proceed with a problem relating ages in the present and the future.

Example 2

This year, the age of a father is 4 times the age of his son. Sixteen years later, the age of the father is only twice the age of the son. What is their age difference? What are the current ages of the father and the son?

Solution

In Figure 3, we construct bar models for the ages of the father and the son aligned to the "Now" line and a line 16 years later. We choose the age of the son now to be the unit U.

Next we construct a fraction bar model to show that the age of father is twice the age of his son 16 years later.

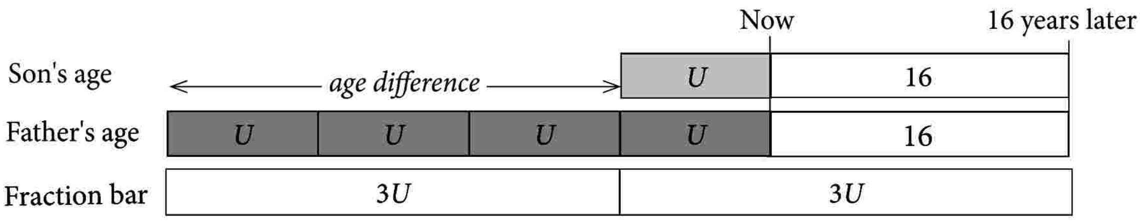

Figure 3

From the bar models, we have $3U = U + 16$ and so $U = 8$.

Their age difference is $3U = 24$ years.

Hence, the current ages of the father and the son are 32 (= 4U) and 8 (= U) respectively.

Now , we recall a *common unit procedure* for integral values.

If an integer x is a multiple of two relatively prime positive integers m and n (i.e. g.c.d $(m, n) = 1$), then we can construct a comparison simple bar model consisting of m and n bars in terms of a common unit U where U is a positive integer such that

$$x = mnU$$

as shown in Figure 4.

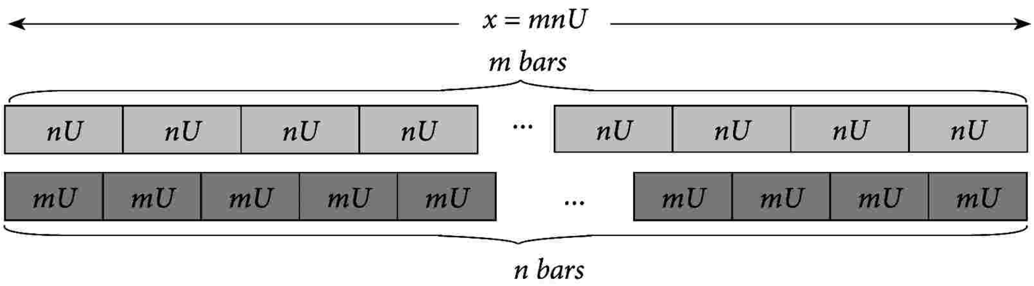

Figure 4

The common unit procedure is applied to the age difference in the next two problems.

Example 3

The age of Lily is $\frac{2}{5}$ of the age of her father. 20 years later, her age is $\frac{3}{5}$ of the age of her father. How old is Lily now?

Solution

In Figure 5,we construct two simple bar models for the fraction $\frac{2}{5}$ and add an additional bar for the situation 20 years later.

Next, we add a fraction bar model with 5 equal parts to relate the fraction $\frac{3}{5}$.

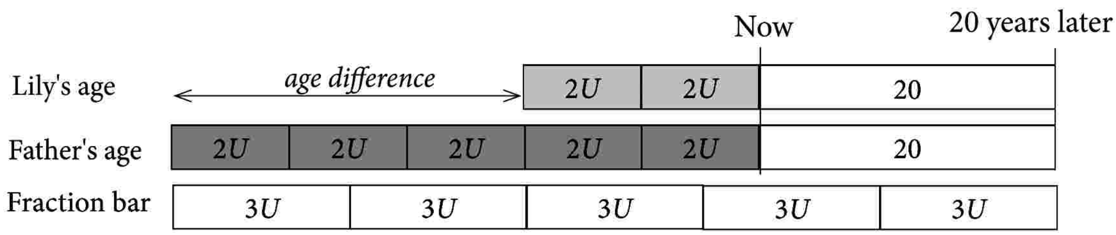

Figure 5

Applying the common unit procedure for the *age difference* in the bar models, there is a common unit U for the bars in the bar models above.

Considering the age of Lily after 20 years, we have

$$2\times 2U + 20 = 3\times 3U,$$

$$U = 4.$$

Lily's age now is 16 ($= 4U$) years old.

Note that the *age difference* between Lily and her father is always 24 ($= 6U$) years. That means Lily was born when her father was 24 years old.

Example 4
Today, the ratio of the age of a man to that of his grandson is 4 : 9 and the grandson is more than 25 years old. Some years ago on the same day , the ratio of the age of a man to that of his grandson was 2 : 5. Today, what is the present age of the man?
(Assume that the age of a person is less than 100.)
Solution
In Figure 6, we construct a comparison bar model for each of the age-situations. Applying the common unit procedure for the 3 bars and 5 bars of the *age difference*, we have a common unit U in the bar models and *age difference* = $3\times5U$, where U is some positive integer.

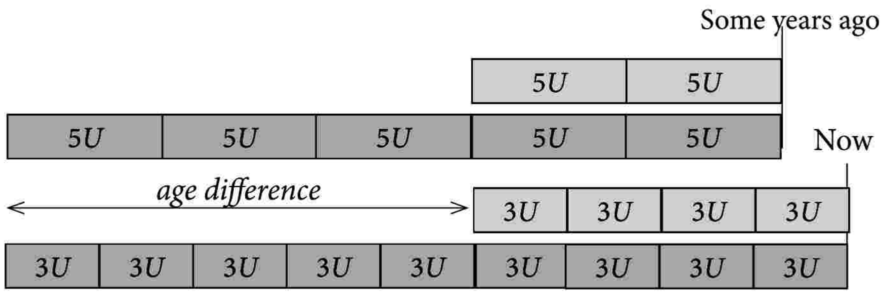

Figure 6

Today the age of the grandson is $4\times3U = 12U$ and the age of the man is $9\times3U= 27U$.
Since $12U > 25$ and $27U < 100$, we have $U = 3$.
Today the age of the man is $27U = 81$.

Note that 3 and 5 are relatively prime.

Age Problems with implicit situations.
Next, we deal with a problem that involves implicit situations.

Example 5
The total age of a father, mother and son now is 90. The father is 5 years older than the mother. Twelve years ago, the total age of the family is 57. What are their ages now?
Solution
12 years ago, the total age of the family decreased by $90 - 57 = 33 = 2\times12 + 9$. Since the total age of the father and mother decreased by $2\times12 = 24$, the age of the son decreased by 9, as shown in Figure 7.

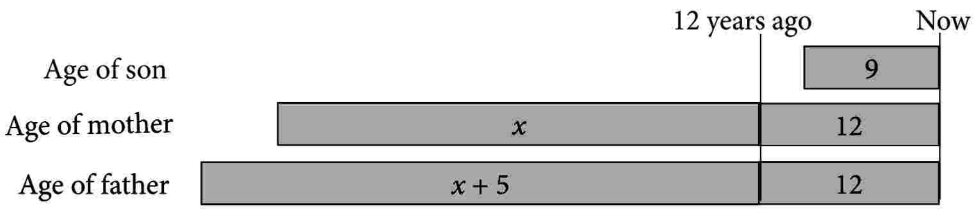

Figure 7

Suppose the age of the mother 12 years ago is x. Since the total age of the father and mother is 57,

$$x + (x + 5) = 57$$
$$x = 26$$

Now, the age of the son is 9, the age of the mother is 38 ($=x + 12$) and age of the father is 43.

Note : From the solution, we can see that bar models help us visualise the implicit situation that the son was born only 9 years ago.

Finally, we will show you how a bar modelling approach helps to visualise implicit situations deduced from the given statements.

Example 6

Bob told Andy : "When I was at your age, you were only 8 years old". Andy told Bob " When I am at your current age, you will be 62 years old". What are their ages now?

Solution

Suppose that Andy is x years old now and so Bob is $(x - 8)$ years older than Andy. That means the *age difference* is $x - 8$.

The two statements in the problem imply the following situations.

Andy is 8 years old when Bob is x years old. $(x - 8)$ years ago

Andy is x years old when Bob is $x + (x - 8)$ years old. Now (Current ages)

Andy is $x + (x - 8)$ years old when Bob is $x + (x - 8) + (x - 8)$ years old. $(x - 8)$ years later

In Figure 8, the bar models depict these situations.

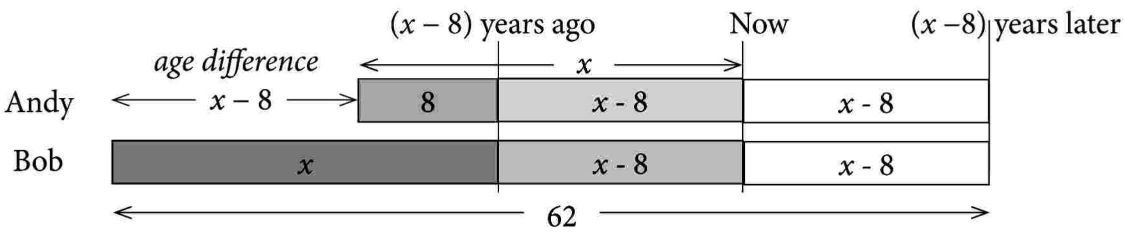

Figure 8

From the above bar models, Bob is 62 years old after $(x - 8)$ years later. Hence

$$x + (x - 8) + (x - 8) = 62,$$
$$3x = 78,$$
$$x = 26$$

So Andy is now 26 years old and Bob is 44 years old.

Remarks

Bar models help to visualise given situations in word problems and relate unknowns effectively for problem solving. However, pupils need to understand the key features of given problems and use appropriate techniques to construct bar models to illustrate the related situations.

Acknowledgement

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References

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