Problem 3

Three robots start together from the same point and travel in the same direction around a circular track of circumference 300m, at the rates of 21, 19, and $^{53}/_{3}$ m/sec respectively. When and where will all three be next together again?

Prize

One $50 book voucher

CONTEST

1. Prizes in the form of book vouchers will be awarded to the first received best solution submitted by secondary school or junior college students in Singapore for each of these problems.

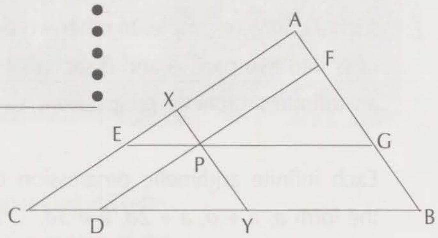
2. To qualify, secondary school or junior college students must include their full name, home address, telephone number, the name of their school and the class they are in, together with their solutions.

3. Solutions should be typed and sent to:
The Editor
Mathematics Medley
c/o Department of Mathematics
National University of Singapore
Kent Ridge, Singapore 119260;
and should arrive before 31 December 1995.

4. The Editor's decision will be final and no correspondence will be entertained.

Problem 4

P is a point in an arbitrary triangle ABC. Through P three lines are drawn parallel to the three sides of triangle ABC as shown in the figure. Given that $EG:CB = h$ and $FD:AC = k$. Find the ratio $XY:AB$.



Prize

One $100 book voucher

Problems Corner

Solutions

to the problems in Volume 22, March 1995

Solution to Problem 1

(i) Solution by Chan Ti Eu, Duman High School, Class 4L.

The answer is any integer n 2. To see this just observe that $x = n - 1$ and $y = n(n - 1)$ is a solution.

(ii) Solution by the Editor.

The answer is any prime number n.
To see this let x, y be positive integers that satisfy $1/x - 1/y = 1/n$. This implies that $n > x$.

Let $n = x + k$ where $1 \quad k \quad n - 1$.
We have $1/y = 1/x - 1/n = 1/(n - k) - 1/n = \frac{k}{n(n-k)}$ and hence $y = \frac{n(n-k)}{k}$.

The case $k = 1$ corresponds to the solution in (i).

Therefore the equation will have more than one solution in positive integers x and y precisely when $n(n - k)$ is a multiple of k for $2 \quad k \quad n - 1$. Now $n(n - k) = n^2 - nk$ is a multiple of k precisely when n^2 is a multiple of k. And when n^2 is a multiple of k for $2 \quad k \quad n - 1$, n itself must be a composite number. Thus for example when $n = 4$, the solutions are $x = 3$, $y = 12$ and $x = 2$, $y = 4$.

Hence for the given equation to have a unique solution, n must be a prime number, and the solution is $x = n - 1$, $y = n(n - 1)$ as mentioned above.

Solution to Problem 2

Solution by the Editor.

The answer is Brett, Calvin and David started with 94, 47 and 141 dollars respectively.

To see this let A, B, C be the amounts of money that David, Brett and Calvin started with and x, y, z the amounts they had respectively at the beginning of the last hand. Therefore $A = 3C$, $B = 2C$ and the last pot contained $x/2 + y/3 + z/6$. At the end of the game, we have

$$x/2 + 1/3(x/2 + y/3 + z/6) = A = 3C,$$
$$2y/3 + 1/3(x/2 + y/3 + z/6) = B = 2C,$$
$$5z/6 + 1/3(x/2 + y/3 + z/6) = C.$$

Solving for x in terms of C, we have $x = \frac{198}{47}C$.
Since x is an integer and 47 is a prime which does not divide 198, we have $C = 47k$ for some positive integer k. Therefore $A = 141k$, $B = 94k$, $C = 47k$ and $x = 198k$.

Since at any round the money put into the pot by each player is the product of two positive integers not larger than 13, we have $x/2 \quad 13^2$. Hence $198k/2 \quad 169$ and so we must have $k = 1$.

Editor's note: *No complete solutions have been received. Chan Ti Eu was awarded a consolation prize of $30 book voucher.*