Problems Corner

CONTEST

Prizes in the form of book vouchers will be awarded to the first received best solution submitted by secondary school or junior college students in Singapore for each of these problems.

To qualify, secondary school or junior college students must include their full name, home address, telephone number, the name of their school and the class they are in, together with their solutions.

Solutions should be typed and sent to:

The Editor, Mathematical Medley
c/o Department of Mathematics
National University of Singapore
Kent Ridge
Singapore 119260

and should arrive before ***30 June 1997.***

The Editor's decision will be final and no correspondence will be entertained.

Problem 1

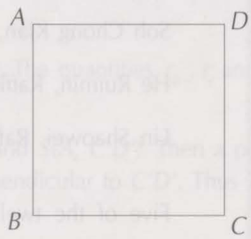
Let a and b be two non-zero integers such that $|a| \le 400$ and $|b| \le 399$. Prove that

$$|a\sqrt{3} + b\sqrt{8}| > \frac{1}{1997}$$

Prize

One $100 book voucher

Problem 2



In the figure $ABCD$ is a square. Given that there is a point P on the same plane such that $PA = 3$, $PB = 7$ and $PD = 5$. Find the area of $ABCD$ when

(a) P is inside $ABCD$;

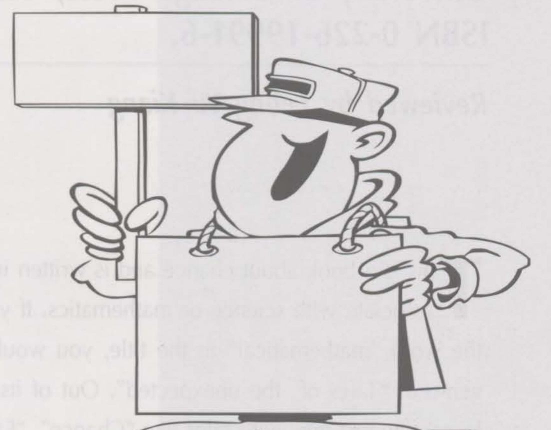
(b) P is outside $ABCD$.

Prize

One $100 book voucher

Solutions

to the problems in Volume 23, No. 2 September 1996



Problem 3

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be four polynomials in x which satisfy the identity

$$P(x^4) + xQ(x^4) + x^2R(x^4) \equiv (x^3 + x^2 + x + 1)S(x).$$

Prove that $(x - 1)$ is a common factor of $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

Solution to Problem 3

by Ng Beow Leng, Nanyang Girl's High School, Class 4S4.

By substituting $x = 1, -1, i, -i$ separately into the given equation, we have

$$P(1) + Q(1) + R(1) = 4S(1) \quad (1)$$
$$P(1) - Q(1) + R(1) = 0 \quad (2)$$
$$P(1) + iQ(1) - R(1) = 0 \quad (3)$$
$$P(1) - iQ(1) - R(1) = 0 \quad (4)$$

Solving simultaneously equations (1) — (4) gives $P(1) = Q(1) = R(1) = S(1) = 0$. Thus $(x - 1)$ is a common factor of $P(x)$, $Q(x)$, $R(x)$ and $S(x)$.

Solved also by Chan Tian Heong, The Chinese High School, Class 4D; He Ruimin, Raffles Institution, Class 3J; G. Venkateswara Rao, Hwa Chong Junior College, Class 96S32; Soh Chong Kian, The Chinese High School, Class 3A; Tan Jit Hin, Raffles Junior College, Class 1S01A. Two incorrect solutions were received.

Editor's note: The \$100 prize was shared equally by Chan Tian Heong, He Ruimin, Ng Beow Leng, G. Venkateswara Rao and Tan Jit Hin.

Problem 4

Prove that

$$1 + \frac{1}{1996}\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{1996}\right) > (1997)^{\frac{1}{1996}}$$

Solution to Problem 4

by Eric Fang Kin Meng, RJC, Class 1S01C

$$1 + (1/1996)(1 + 1/2 + 1/3 + \dots + 1/1996)$$
$$= (1/1996)[(1 + 1) + (1 + 1/2) + \dots + (1 + 1/1996)]$$
$$= (1/1996)(2 + 3/2 + 4/3 + \dots + 1997/1996) > (2 \cdot 3/2 \cdot 4/3 \dots 1997/1996)^{1/1996}$$
$$= (1997)^{1/1996}$$

The above inequality was proven by direct application of the Arithmetic Mean-Geometric Mean inequality. Equality does not hold as the elements are non-equal.

Solved also by Chan Tian Heong, The Chinese High School, Class 4D; Ng Beow Leng, Nanyang Girls' High School, Class 4S4; Soh Chong Kian, The Chinese High School, Class 3A.

Editor's Note: The \$100 prize was shared equally by Chan Tian Heong, Eric Fang Kin Meng, Ng Beow Leng and Soh Chong Kian.