

# What is $1 + 2 + 3 + \dots$ ?

For a positive integer  $n$ , define

$$S_n = 1 + 2 + \dots + n$$

The reader is invited to prove:

**Theorem** *For any positive integer  $N$ , there is a positive integer  $k$  such that  $S_n \geq N$  for every  $n \geq k$ .*

A suitable  $k$  is needed for any given  $N$ . For  $N = 3$ ,  $k = 2$  works, but so does any larger integer. Your choice of  $k$  does not need to be the smallest possible.

The theorem affirms our intuition that the sequence  $S_1, S_2, S_3, \dots$  increases without any limit. Hence the following statement:

$$1 + 2 + 3 + \dots = \infty$$

In words, the sum of the positive integers is infinity.

The equal sign has a universal meaning in arithmetic, which extends convincingly to the statement above. It is irresponsible, even dangerous, to argue that  $1 + 2 + 3 + \dots$  is anything else. Worse, if the person is mathematically accomplished, for the audience might blame their school teachers. If it is indeed important to do so, then use a new symbol. For instance,

$$1 + 2 + 3 + \dots \quad [!?!] \quad - \frac{1}{12}$$

might even make a good T-shirt.